



# Compositional Verification of First-Order Masking Countermeasures against Power Side-Channel Attacks

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Power side-channel attacks allow an adversary to efficiently and effectively steal secret information (e.g., keys) by exploiting the correlation between secret data and runtime power consumption, hence posing a serious threat to software security, particularly cryptographic implementations. Masking is a commonly used countermeasure against such attacks, which breaks the statistical dependence between secret data and side-channel leaks via randomization. In a nutshell, a variable is represented by a vector of shares armed with random variables, called *masking encoding*, on which cryptographic computations are performed. While compositional verification for the security of masked cryptographic implementations has received much attention because of its high efficiency, existing compositional approaches either use implicitly fixed pre-conditions that may not be fulfilled by state-of-the-art efficient implementations, or require user-provided hard-coded pre-conditions that are time consuming and highly non-trivial, even for an expert. In this article, we tackle the compositional verification problem of first-order masking countermeasures, where first-order means that the adversary is allowed to access only one intermediate computation result. Following the literature, we consider countermeasures given as gadgets, which are special procedures whose inputs are masking encodings of variables. We introduce a new security notion parameterized by an explicit pre-condition for each gadget, as well as composition rules for reasoning about masking countermeasures against power side-channel attacks. We propose accompanying efficient algorithms to automatically infer proper pre-conditions, based on which our new compositional approach can efficiently and automatically prove security for masked implementations. We implement our approaches as a tool MASKCV and conduct experiments on publicly available masked cryptographic implementations including 10 different full AES implementations. The experimental results confirm the effectiveness and efficiency of our approach.

CCS Concepts: • **Security and privacy** → **Side-channel analysis and countermeasures**; • **Software and its engineering** → **Software verification**; **Automated static analysis**;

Additional Key Words and Phrases: Formal verification, compositional verification, cryptographic programs, side-channel attacks, masking countermeasures

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**1 INTRODUCTION**

Cryptographic programs have become an indispensable part of contemporary software systems. Practical implementations of cryptographic algorithms unfortunately suffer from side-channel attacks which exploit the statistical dependence between the secret (e.g., keys) and side-channel information (e.g., power consumption [72] and execution time [71]) to effectively recover the secret. In this article, we focus on power side-channel attacks which have been shown to be successful in practice. For instance, DES [35, 72], AES [84, 94, 97], RSA [56], elliptic curve cryptography [36, 64, 74, 80], and post-quantum cryptography [67, 86, 89] have been the victims. Essentially, power side-channel attacks exploit the correlation between the runtime power consumption of a device executing cryptographic implementations and the secret. The adversary can effectively deduce the results of chosen intermediate computations by statistically analyzing the power consumption traces based on the fact that the power consumption of CMOS transistors (e.g., register) for storing signals 1 and 0 is typically different. When the deduced intermediate computation results rely upon the secret, the adversary can further infer the value of the secret from the intermediate computations and their results. For example, consider the statement  $x = k \oplus p$ , where  $k$  is the secret,  $p$  is some public input, and  $\oplus$  is the exclusive-OR operation. Obviously, the value of  $x$  depends on the value of  $k$  for any fixed value of  $p$ . For instance, fixing  $p = 0$ , the probability of  $x = 1$  is 100% if  $k = 1$  and the probability of  $x = 0$  is 100% if  $k = 0$ . Moreover, the power consumption of a device executing this statement is correlated with the value of  $x$  (e.g., the power consumption of the CMOS transistor like the register for storing  $x$  varies with the value of  $x$ ). As a result, the adversary can choose  $p$ , and statistically analyze the power consumption traces to deduce the value of  $x$ , which leads to the disclosure of the secret  $k$  by  $x \oplus p$ . To effectively and efficiently deduce the secret in real-world cryptographic implementations via power side-channel information, the adversary often chooses invertible intermediate computations, such as input or output of Sbox in AES, so that the secret can be quickly recovered from these intermediate computations and their results.

Masking is an effective countermeasure against power side-channel attacks, aiming at breaking the statistical dependence between the secret and power consumption via randomization [63]. Typically, an order- $d$  masking scheme splits the secret into  $d + 1$  shares such that the joint distribution of any  $d$  shares is (statistically) independent of the secret, thus the adversary cannot infer any information of the secret by observing the values of any  $d$  shares via power side channels. For example, the secret  $k$  can be split into two shares  $k \oplus r$  and  $r$  via first-order masking, where  $r$  is a uniformly sampled value, and  $(k \oplus r, r)$  forms a masking encoding of the secret  $k$ . Since the distributions of both  $k \oplus r$  and  $r$  are independent of the secret  $k$ , the adversary cannot infer the information of the secret  $k$  by observing the value of  $k \oplus r$ . In contrast, the distribution of  $k \wedge r$  depends upon the secret  $k$ , namely the probability of  $k \wedge r$  being 1 is 50% if  $k = 1$  but is 0% if  $k = 0$ . The adversary can infer the value of  $k$  by observing the value of  $k \wedge r$ . The main challenge is to develop an efficient and secure implementation of each cryptographic algorithm  $f$  using masking that performs computations on the secret shares of the original input  $x$ , and produces output shares from which the desired output  $f(x)$  can be recovered. As efficiency is a major concern in, for example, resource-limited devices [20], various new masked implementations for

finite-field multiplication, a key building block to implement most cryptographic algorithms, have been proposed recently [8, 11, 14, 15, 26, 58, 68, 93]. These more recent implementations require less randomness and/or operations than the original one proposed by Ishai et al. [63].

Clearly, a masked implementation must meet some security requirement, such as the *order- $d$  probing security* [63] which asserts that the joint distribution of any  $d$  observable variables to the adversary should be independent of the secret data. For instance, the masked implementation of the statement  $x = k \oplus p$  via order- $d$  masking is  $r_{d+1} = k \oplus r_1 \oplus \dots \oplus r_d$ ;  $x = r_{d+1} \oplus p$ , where  $r_1, \dots, r_d$  are uniformly sampled values,  $(r_1, \dots, r_d, r_{d+1})$  forms a masking encoding of the secret  $k$ , and the desired result  $k \oplus p$  can be obtained from the output encoding  $(r_1, \dots, r_d, x)$  via computing  $r_1 \oplus \dots \oplus r_d \oplus x$ . This masked implementation is order- $d$  probing secure because the joint distribution of any  $d$  observable variables from  $\{r_1, \dots, r_d, r_{d+1}, x\}$  is uniform, thus independent of the secret  $k$ . However, it is *not* order- $(d + 1)$  probing secure, because the joint distribution of some  $(d + 1)$  observable variables (e.g.,  $\{r_1, \dots, r_d, r_{d+1}\}$  or  $\{r_1, \dots, r_d, x\}$ ) depends on the secret  $k$ . Indeed,  $r_1 \oplus \dots \oplus r_d \oplus r_{d+1}$  is the same as the secret  $k$  and  $r_1 \oplus \dots \oplus r_d \oplus x$  is the same as  $k \oplus p$  while  $p$  is known to the adversary. However, designing masked implementations is labor intensive and error prone. Indeed, some published masked implementations [87, 90] were later shown to be vulnerable against power side-channel attacks under the same leakage model and masking order [40, 41]. To address this concern, various formal verification approaches have been proposed, which form an important subarea of software security research. These approaches can be largely be divided into two categories: non-compositional (i.e., intra-procedural) and compositional (i.e., inter-procedural) approaches [9, 18, 53]. Following the naming convention in the literature on masked implementations of cryptographic algorithms, procedures are called *gadgets* in this work, because the input parameters and return value of the procedures are often masking encodings. Furthermore, a gadget is referred to as *simple gadget* if it does not contain any call statements, and otherwise it is referred to as *composite gadget*.

Non-compositional approaches take a simple gadget as input and verify the observable variables one by one or set by set. The underlying techniques include symbolic analysis [21, 38, 76, 77, 81, 91, 92], SAT/SMT-based analysis [23, 47–50], BDD analysis [69], and hybrid approaches [52, 54, 55, 98]. Note that other works [21, 23, 47–50, 55, 69, 98] focus exclusively on the bitwise logical operations (e.g., and, or, exclusive-or) and cannot directly handle *arithmetic* operations (e.g., addition, subtraction).

To verify a composite gadget using non-compositional approaches, all the gadget calls must be inlined first. However, inlining gadget calls will produce plenty of observable variables with large computation expressions, leading to the inefficient verification. Compositional approaches are thus proposed by leveraging *stronger* security notions [9, 10, 12, 16–19, 25, 26, 30, 31, 69] to directly analyze composite gadgets without inlining. All of these approaches implicitly specify a fixed pre-condition for each gadget that intuitively restricts the number or positions of input shares. Unfortunately, many modern efficient implementations (e.g., [14, 20]) do not satisfy these implicitly imposed pre-conditions, creating a gap between formally provable probing security and practically used (efficient) masked implementations. To address this issue, an assume-guarantee-based compositional approach has been proposed [53] for verifying first-order security of arithmetic masked programs. However, hard-coded (non-standard) pre-conditions, specifying the relation between support variables of the computations of formal arguments must be provided by users, which impedes fully automated compositional verification. While inferring specifications and contracts of procedures from source code is a well-researched area in formal safety verification, e.g., [3, 5, 62], they are not applicable to compositional verification of security for masked implementations.

In this work, we fill this gap for first-order probing security, which is the focus of a large body of research work, particularly in software security [47–50, 53, 54, 76, 77, 81, 91, 92, 98].

Following the literature, we consider masked implementations that are given as gadgets and composition of gadgets, which are special procedures whose inputs are masking encodings of variables. Specifically, we propose a new security notion parameterized by an explicit pre-condition  $\mathbb{I}$  for each gadget, called  $\mathbb{I}$ -*Non-Interference*, based on which we present a compositional verification approach that is applicable to those efficient masked implementations without user-provided pre-conditions. The pre-condition  $\mathbb{I}$  of a gadget  $g$  is a set of variable sets, and is used to characterize the first-order probing security of the gadget  $g$ , namely each observable variable of the gadget  $g$  can be perfectly simulated by a variable set from  $\mathbb{I}$ . Note that the pre-condition  $\mathbb{I}$  can vary with gadgets. For some gadgets, the formal parameters (i.e., input shares) are sufficient to simulate the internal variables; otherwise, internal variables would be added into  $\mathbb{I}$ . (The worst case would be that all the internal variables are added into  $\mathbb{I}$ , which amounts to inlining the gadget call.) The flexibility of  $\mathbb{I}$  gives wider applicability to generic and efficient masked implementations while fixed pre-conditions [9, 10, 12, 16–19, 25, 26, 30, 31, 69] fail.

Figure 1 shows an overview of our method, consisting of two main steps: preprocessing and verification. Given a masked program and its security type annotation of input parameters, the preprocessing step automatically unfolds loops in the program and transforms the program into its intermediate representation in the **Static Single Assignment (SSA)** form. The verification step infers pre-conditions of gadgets based on which first-order probing security is verified. We propose efficient algorithms for inferring pre-conditions of both simple and composite gadgets. The pre-conditions of simple gadgets are saturated during judgment inference via a sound proof system until they become sufficient. The pre-conditions of composite gadgets are inferred by leveraging a novel composition rule that composes pre-conditions of the called gadgets. More specifically, the verification process starts by inferring the pre-condition  $\mathbb{I}$  for the main gadget. The gadget call statements in the main gadget are iteratively traversed during which the pre-conditions of the called gadgets are computed by recursively calling the algorithms for inferring pre-conditions of simple and/or composite gadgets. To facilitate the processing of the subsequent gadget call statements, the resulting pre-conditions of gadgets are cached. Finally, the pre-conditions of called gadgets are composed according to the composition rule, resulting in the pre-condition  $\mathbb{I}$  for the main gadget. Finally, to check whether a program is first-order probing secure or not (i.e., each observable variable is statistically independent of the secret inputs), we can check the pre-condition  $\mathbb{I}$  of the main gadget, where the pre-condition  $\mathbb{I}$  is expected to be much smaller than the size of observable variables after inlining. Moreover, we observe that a variable (called *dominated variable*) whose computation is perfectly masked by a random variable (called *dominant variable*) could also be used to mask other variables. For instance, the variable  $x$  with  $x = k \oplus r$  is perfectly masked by the random variable  $r$  as the distribution of  $x$  is uniform for any fixed value of  $k$ , thus  $x$  is a dominated variable and  $r$  is its dominant variable. This observation often allows us to quickly conclude that any dominated variable is statistically independent of the secret inputs, and thus is leveraged in the algorithms for inferring pre-conditions of simple and composite gadgets to reduce the size of pre-conditions and improve verification efficiency.

We implement our approach as a tool `MASKCV` and conduct extensive experiments on several publicly available masked implementations [53] including 10 different masked implementations of full AES. The experimental results confirm the efficacy of our approach (e.g., `MASKCV` can prove each of the 10 masked implementations of full AES in no more than 0.04 seconds). Compared with the compositional approach `QMVÉRIF` [53], our approach performs significantly better when no pre-conditions are provided to `QMVÉRIF` and achieves competitive efficiency when all the user-defined pre-conditions are provided to `QMVÉRIF`. Indeed, `QMVÉRIF` takes 197 to 3,505 seconds to prove each of 10 masked implementations of full AES when no pre-conditions are provided. (To the best of our knowledge, `QMVÉRIF` is the only compositional tool that can

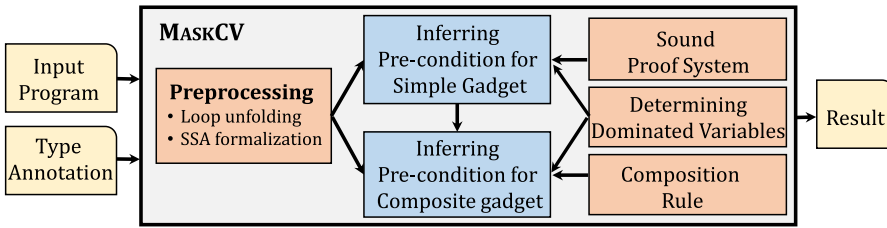


Fig. 1. Overview of our method.

verify these benchmarks.) Compared with the state-of-the-art non-compositional approaches SILVER [69] and LeakageVerif [77], our approach is significantly more efficient and effective. For instance, SILVER does not support masked implementations of full AES which involve arithmetic operations and runs out of time or memory (6 hours, 256 GB of RAM) on two Boolean implementations of AES Sbox which can be proved by MASKCV in 0.01 seconds; LeakageVerif runs out of time or memory on 6 out of 10 masked implementations of full AES and takes at least 559 seconds for any of the other four masked implementations.

The main contributions of this work are summarized as follows:

- We introduce a new security notion, called  $\mathbb{I}$ -Non-Interference, which is parameterized by an explicit and variable pre-condition  $\mathbb{I}$ . We also present a composition rule for compositional reasoning about first-order probing security of efficient masked implementations that cannot be handled by existing fully automated compositional verification approaches [9, 10, 12, 16–19, 25, 26, 30, 31, 69].
- We propose efficient algorithms for inferring pre-conditions of both simple and composite gadgets, leading to an efficient and fully automated composition verification approach for first-order probing security of arithmetic masked programs. Thus, providing hard-coded (non-standard) pre-conditions in the assume-guarantee-based compositional approach [53], which is time consuming and highly non-trivial even for an expert, can be avoided.
- We propose an efficient approach to determine dominated variables that can reduce the size of pre-conditions and thus significantly improves the verification efficiency. It paves the way for integrating the power side-channel security verification into the software development process and continual verification during development.
- We implement the proposed techniques in a tool MASKCV and conduct extensive experiments on publicly available cryptographic benchmarks, which confirm the effectiveness and efficiency of our approach. In particular, on the 10 masked implementations of full AES, the automatically determined dominated variables improve the verification efficiency by 4 orders of magnitude, and MASKCV shows almost 5 to 6 orders of magnitude improvement with respect to LeakageVerif and QMVERIF without user-defined pre-conditions.

Our work is important for the software engineering community. First, it can be readily used by cryptographers and software developers of cryptographic algorithms to ensure the (first-order) probing security of their implementations against power side-channel attacks. Cryptographic algorithms have been extensively used in various applications, ranging from Blockchain, Internet of Things, and edge computing to smart devices. It is important to ensure that software systems are robust against power side-channel attacks, particularly for security-critical applications, where our work would play a crucial role. Moreover, MASKCV is highly automated and can be fully integrated into the software development process, so it potentially would serve a larger group of software developers. Furthermore, the verification techniques we use, such as a proof system and



Table 1. Notations

Notation	Description	Notation	Description
$\mathbb{I}$	set of variable sets used as pre-condition	$\mathbb{F}$	underlying domain $\{0, \dots, 2^n - 1\}$
$x$	scalar variable	$\vec{x}$	vector of variables used as masking encoding of $x$
$\vec{x}[i]$	the $i$ -th share of the encoding $\vec{x}$	$X_k$	set of private variables of a program
$G_c/G_s/G_{en}$	simple/composite/encoding gadget	$X_p$	set of public variables of a program
$g_{in}/P_{in}$	inlined version of the gadget $g$ /program $P$	$\mathcal{E}(x)$	computation of a variable $x$
$\text{Sub}(x)$	set of the sub-expressions of $\mathcal{E}(x)$	$\text{Var}(\mathcal{E}(x))$	set of support variables of $\mathcal{E}(x)$
$X_{en}^g$	set of input encodings of the gadget $g$	$X_{en}^g[i]$	the $i$ -th input encoding of the gadget $g$
$T_g$	summary of the gadget $g$	$\eta$	valuation for a set of variables
$X_r^g$	set of random variables defined in the gadget $g$	$\mathcal{D}(V)$	set of the joint distributions of variables in $V$
$X_o^g$	set of output variables defined in the gadget $g$	$\text{DomR}(x)$	set of random dominant variables of the variable $x$
$X_a^g$	set of formal parameters of the gadget $g$	$\ \mathcal{E}(x)\ _\eta^p$	probability distribution of $x$ in program $P$ under $\eta$
$X^g$	set of internal variables defined in the gadget $g$	$\ X\ _\eta^p$	joint distribution of variables of $X$ in $P$ under $\eta$

compositional reasoning, represent the state-of-the-art research in quality assurance—an important area of software engineering.

**Outline.** The article is organized as follows. Section 2 gives the preliminary of this work (i.e., cryptographic programs considered in this work, threat model, leakage model, and security notions) and a running example for demonstration. We propose the new security notion  $\mathbb{I}$ -Non-Interference and its composition rule in Section 3. In Section 4, we introduce the concept of dominated variables, an approach to determine dominated variables and the application of dominated variables for inferring pre-conditions. Section 5 presents the algorithms for inferring pre-conditions for both simple and composite gadgets. Section 6 reports our experimental results on various publicly available masked cryptographic implementations. We discuss related work in Section 7 and conclude the article in Section 8. The source code of our tool, raw data of experimental results, and benchmarks are available on Figshare [1].

## 2 PRELIMINARY

In this section, we first introduce the syntax and semantics of the program considered in this work, then recap the threat model, leakage model, and related security notions, and finally present a running example for demonstration of our approach.

**Notations.** For convenient reference, we summarize the notations in Table 1.

### 2.1 Cryptographic Programs

Given a positive integer  $n$ , let  $\mathbb{F}$  be the domain  $\{0, \dots, 2^n - 1\}$  (e.g., the finite-field  $\mathbb{GF}(2^n)$ ). We use the syntax shown in Figure 2 to describe masked implementations. Symbols such as  $x, a, a_1, a_m$  are scalar variables, and symbols with arrowed overline such as  $\vec{b}, \vec{a}_1, \vec{a}_m$  represent vectors of variables. A vector  $\vec{x}$  of variables is used to represent the shares of a scalar variable  $x$ , thus called a (masking) *encoding* of the scalar variable  $x$  in this work. We denote by  $\vec{x}[i]$  the  $i$ -th variable in the encoding  $\vec{x}$ , which is one share of the scalar variable  $x$  and by  $\bigoplus \vec{x}$  the XOR of all the shares of the encoding  $\vec{x}$ . A masked implementation always has a security parameter  $d$  denoting order- $d$  masking, and the size of each encoding is typically  $d + 1$ . We explain the syntax rules in Figure 2 from bottom to top.

A program is defined as a sequence of gadget definitions with a main gadget, where scalar variables  $a_1, \dots, a_m$  are its formal parameters. We assume that the formal parameters of the main gadget are classified into their types, either private or public, provided by the users. We denote by  $X_k$  the set of private variables and  $X_p$  the set of public variables so that  $X_k \cup X_p = \{a_1, \dots, a_m\}$ . The main gadget has a sequence of (masking) encoding call statements `enstmt` and gadget call statements `gstmt`, ending with a return statement `return  $\vec{b}$`  which gives the output encoding  $\vec{b}$ .

Constant:	$\mathbb{F} \ni c$	::=	$n$ -bit constant
Operation:	$\text{Op} \ni \circ$	::=	$\wedge \mid \vee \mid \oplus \mid - \mid + \mid \times \mid \odot$
Expression:	$e$	::=	$c \mid x \mid \neg e \mid e \ll c \mid e \gg c \mid e \circ e$
Statements:	stmts	::=	$x = e; \mid r = \$; \mid \text{stmts}^+$
Simple gadget:	$G_s$	::=	$g(\vec{a}_1, \dots, \vec{a}_m)$ {stmts return $\vec{b}$ ;} $g(\vec{a}_1, \dots, \vec{a}_m)$
Composite gadget:	$G_c$	::=	{gstmt <sup>+</sup> return $\vec{b}$ ;} $g(\vec{a}_1, \dots, \vec{a}_m)$
Gadget call:	gstmt	::=	$\vec{x} = g(\vec{y}_1, \dots, \vec{y}_m);$
Encoding gadget:	$G_{en}$	::=	$f(a)\{\text{stmts return } \vec{a};\}$
Encoding call:	enstmt	::=	$\vec{x} = f(x);$
Program:	$P$	::=	$G_{en}^+ G_s^+ G_c^*$ main( $a_1, \dots, a_m$ ) {enstmt <sup>+</sup> gstmt <sup>+</sup> return $\vec{b}$ ;} $G_{en}^+ G_s^+ G_c^*$

Fig. 2. Syntax of programs.

An encoding call statement (i.e., enstmt) generates an encoding  $\vec{x}$  of a scalar variable  $x$  by calling an encoding gadget (i.e.,  $\vec{x} = f(x)$ ). An encoding gadget (i.e.,  $f(a)\{\text{stmts}; \text{return } \vec{a};\}$ ) is defined as a common procedure, which takes a scalar variable  $a$  as input and outputs the encoding of  $a$  (i.e.,  $\vec{a}$ ). An encoding gadget computes the desired encoding via a sequence of statements that can express different encoding schemes. Normally, the first  $d$  variables in an encoding (i.e.,  $\vec{a}[1] \dots \vec{a}[d]$ ) are generated by random sampling, whereas  $\vec{a}[d+1]$  is a computation of  $\vec{a}[1], \dots, \vec{a}[d]$  and the input variable  $a$ . *Boolean masking* is always used in implementing masked cryptographic algorithms which only use bitwise logical operations (see the following) and  $\vec{a}[d+1] = \vec{a}[1] \oplus \vec{a}[2] \oplus \dots \oplus \vec{a}[d] \oplus a$ . Masked implementations which only use arithmetic operations are called *arithmetic masking*, and they compute  $\vec{a}[d+1]$  using modular addition, namely  $\vec{a}[d+1] = \vec{a}[1] + \vec{a}[2] + \dots + \vec{a}[d] + a$ . There are some implementations that use both bitwise logical operations and arithmetic operations for which conversions between Boolean and arithmetic masking [37, 39] are used.

A gadget call statement (i.e., gstmt) assigns the result of a gadget call to an internal encoding. The syntax supports two types of gadgets: *simple* (i.e.,  $G_s$ ) and *composite* (i.e.,  $G_c$ ) gadgets, both of which take a list of input encodings  $\vec{a}_1, \dots, \vec{a}_m$  as inputs and return the output encoding  $\vec{b}$ . A simple gadget only contains a sequence of statements (i.e., stmts) without involving any gadget call statements, whereas a composite gadget only contains a sequence of gadget call statements (i.e., gstmt<sup>+</sup>) with unique labels  $\ell$  (denoting call sites), both of which end with a return statement return  $\vec{b}$ . We assume that all the gadgets are given in SSA form. We denote by  $X_{en}^g$  the vector (e.g.,  $[\vec{a}_1, \dots, \vec{a}_m]$ ) of all the input encodings of the gadget  $g$ , and by  $X_{en}^g[i]$  the  $i$ -th input encoding (e.g.,  $\vec{a}_i$ ). Let  $X_a^g$  be the union of all the variables in input encodings of the gadget  $g$  (i.e.,  $X_a^g = \bigcup_{1 \leq i \leq m} \vec{a}_i$ ). If  $g$  is a main gadget or an encoding gadget,  $X_a^g$  is the set of input parameters, which are scalar variables. An encoding gadget may be referred to as a simple gadget as well, because it does not contain any call statements.

There are two types of statements. One is the common assignment statement of the form  $x = e$  which assigns the value of the expression  $e$  to the scale variable  $x$ . The other is of the form  $r = \$$  which assigns a uniformly sampled random value to the scale variable  $r$ , thus  $r$  is a *random variable*. Random variables are used to mask private-related variables. We denote by  $X_r^g$  and  $X^g$  respectively the set of random variables and the set of internal variables (excluding input parameters  $X_a^g$ ) defined in the gadget  $g$ . Similarly, we denote by  $X_o^g$  the set of output variables of the gadget  $g$ .

1	Encoding( $k$ ){	5	XOR( $\vec{a}, \vec{b}$ ){	9	main( $k_1, k_2$ ){
2	$\vec{a}[1] = \$;$	6	<b>for</b> ( $i = 1; i \leq 2; i++$ ){	10	$\vec{a} = \text{Encoding}(k_1);$
3	$\vec{a}[2] = \vec{a}[1] \oplus k;$	7	$\vec{c}[i] = \vec{a}[i] \oplus \vec{b}[i];$	11	$\vec{b} = \text{Encoding}(k_2);$
4	<b>return</b> $\vec{a};$ } // $\oplus \vec{a} = k$	8	<b>return</b> $\vec{c};$ } // $\oplus \vec{c} = \oplus \vec{a} \oplus \oplus \vec{b}$	12	$\vec{c} = \text{XOR}(\vec{a}, \vec{b});$
				13	<b>return</b> $\vec{c};$ } // $\oplus \vec{c} = k_1 \oplus k_2$

Fig. 3. Encoding is an encoding gadget, XOR is a simple gadget, and main is the main gadget.

An expression  $e$  is built up from scale variables and constants with the following operations:

- *Bitwise logical operations*: *and* ( $\wedge$ ), *or* ( $\vee$ ), *exclusive-or* ( $\oplus$ ), *negation* ( $\neg$ ), *left shift* ( $\ll$ ), *right shift* ( $\gg$ ).
- *Modulo  $2^n$  arithmetic operations*: *subtraction* ( $-$ ), *addition* ( $+$ ), *multiplication* ( $\times$ ) for which  $\mathbb{F}$  is regarded as the ring  $\mathbb{Z}_{2^n}$  of integers modulo  $2^n$ .
- *Finite-field operation*: *multiplication* ( $\odot$ ), for which  $\mathbb{F}$  is considered to be the Galois field  $\text{GF}(2^n)$ .

For a simple gadget  $g$ , the *computation*  $\mathcal{E}(x)$  of each variable  $x \in X_a^g \cup X^g$  is defined as follows:

- If  $x \in X_a^g \cup X_r^g$ ,  $\mathcal{E}(x) = x$ .
- If  $x = e$ ,  $\mathcal{E}(x)$  is obtained by recursively replacing all the occurrences of each variable  $y$  in  $e$  by its computation  $\mathcal{E}(y)$  until no more update can be made.

Intuitively,  $\mathcal{E}(x)$  defines the computation of the variable  $x$  in terms of input parameters, constants, and random variables, and thus can be seen as the symbolic value of the variable  $x$ . We denote by  $\text{Var}(\mathcal{E}(x))$  the set of support variables of the variable  $x$ —that is, the input parameters and random variables that are involved in the computation  $\text{Var}(\mathcal{E}(x))$ .

For a composite gadget  $g$ , to obtain the computation of a variable  $x \in X_a^g \cup X^g$ , gadget calls should be inlined first. Inlining a gadget call is the same as inlining a procedure call, which replaces the gadget call by the gadget body of the callee, followed by assignments mimicking the return statement, where the local variables are appended with  $@\ell$  of the call-site  $\ell$  to avoid name conflict. We denote by  $f(\vec{x}_1, \dots, \vec{x}_m)@\ell$  the corresponding statements after inlining the gadget call  $\vec{y} = f(\vec{x}_1, \dots, \vec{x}_m)$  with call-site  $\ell$ . After recursively inlining all the gadget calls, a composite gadget turns to a simple one. We denote by  $g_{in}$  the inlined version of  $g$ . Specifically,  $P_{in}$  is the inlined version of the program  $P$ .

*Example 2.1.* Figure 3 presents three gadgets, where Encoding is an encoding gadget, XOR is a simple gadget [63], and main is the main gadget.

The Encoding gadget splits an input  $k$  into two shares  $\vec{a}[1]$  and  $\vec{a}[2]$  via first-order Boolean masking, where  $\vec{a}[1]$  is randomly sampled and  $\vec{a}[2] = \vec{a}[1] \oplus k$  masks  $k$  by XORing  $\vec{a}[1]$ , resulting in the encoding  $\vec{a}$  of the scalar variable  $k$ . For the Encoding gadget, we have the following:

- $X_a^{\text{Encoding}} = \{k\}$ ,  $X_r^{\text{Encoding}} = \{\vec{a}[1]\}$ ,  $X_o^{\text{Encoding}} = X^{\text{Encoding}} = \{\vec{a}[1], \vec{a}[2]\}$ .
- $\mathcal{E}(k) = k$  and  $\mathcal{E}(\vec{a}[1]) = \vec{a}[1]$  (as  $k, \vec{a}[1] \in X_a^{\text{Encoding}} \cup X_r^{\text{Encoding}}$ ), and  $\mathcal{E}(\vec{a}[2]) = \vec{a}[1] \oplus k$ .

The XOR gadget performs share-wise XOR—that is, it produces the encoding  $(\vec{a}[1] \oplus \vec{b}[1], \vec{a}[2] \oplus \vec{b}[2])$  for the input encodings  $\vec{a}$  and  $\vec{b}$ . We have the following:

- $X_{en}^{\text{XOR}} = \{\vec{a}, \vec{b}\}$ , where  $X_{en}^{\text{XOR}}[1] = \vec{a}$  and  $X_{en}^{\text{XOR}}[2] = \vec{b}$ .
- $X_a^{\text{XOR}} = \{\vec{a}[1], \vec{a}[2], \vec{b}[1], \vec{b}[2]\}$ ,  $X_r^{\text{XOR}} = \emptyset$ ,  $X_o^{\text{XOR}} = X^{\text{XOR}} = \{\vec{c}[1], \vec{c}[2]\}$ .
- $\mathcal{E}(x) = x$  for every  $x \in X_a^{\text{XOR}}$ , and  $\mathcal{E}(\vec{c}[i]) = \vec{a}[i] \oplus \vec{b}[i]$  for  $i = 1, 2$ .



1	$\text{main}_{in}(k_1, k_2)\{$	6	$\vec{a}[1]@11 = \$;$	10	$\vec{c}[1]@12 = \vec{a}[1] \oplus \vec{b}[1];$
2	$\vec{a}[1]@10 = \$;$	7	$\vec{a}[2]@11 = \vec{a}[1]@11 \oplus k_2;$	11	$\vec{c}[2]@12 = \vec{a}[2] \oplus \vec{b}[2];$
3	$\vec{a}[2]@10 = \vec{a}[1]@10 \oplus k_1;$	8	$\vec{b}[1] = \vec{a}[1]@11;$	12	$\vec{c}[1] = \vec{c}[1]@12;$
4	$\vec{a}[1] = \vec{a}[1]@10;$	9	$\vec{b}[2] = \vec{a}[2]@11;$	13	$\vec{c}[2] = \vec{c}[2]@12;$
5	$\vec{a}[2] = \vec{a}[2]@10;$			14	<b>return</b> $\vec{c};$ }

Fig. 4. The inlined version  $\text{main}_{in}$  of the main gadget.

The main gadget consists of two encoding call statements to the Encoding gadget and one gadget call statement to the simple gadget XOR. It takes two secrets  $k_1$  and  $k_2$  as inputs, first computes their encodings via the Encoding gadget, and then computes  $k_1 \oplus k_2$  via the simple gadget XOR using the encodings  $\vec{a}$  and  $\vec{b}$  of  $k_1$  and  $k_2$ , resulting in the output encoding  $\vec{c}$  of  $k_1 \oplus k_2$ . We have the following:

- $X_a^{\text{main}} = X_k = \{k_1, k_2\}, X_p = \emptyset.$
- $X_r^{\text{main}} = \emptyset, X^{\text{main}} = \{\vec{a}[1], \vec{a}[2], \vec{b}[1], \vec{b}[2], \vec{c}[1], \vec{c}[2]\}, X_o^{\text{main}} = \{\vec{c}[1], \vec{c}[2]\}.$

The inlined version of the main gadget is shown in Figure 4. We have the following:

- $X_a^{\text{main}_{in}} = X_k = \{k_1, k_2\}, X_p = \emptyset.$
- $X_r^{\text{main}_{in}} = \{\vec{a}[1]@10, \vec{a}[1]@11\}, X_o^{\text{main}_{in}} = \{\vec{c}[1], \vec{c}[2]\}.$
- $X^{\text{main}_{in}} = X^{\text{main}} \cup \{\vec{a}[1]@10, \vec{a}[2]@10, \vec{a}[1]@11, \vec{a}[2]@11, \vec{c}[1]@12, \vec{c}[2]@12\}.$
- The computations  $\mathcal{E}(x)$  for every  $x \in X^{\text{main}_{in}}$  can be constructed accordingly—for example,

$$\mathcal{E}(\vec{c}[1]) = \vec{a}[1]@10 \oplus \vec{a}[1]@11 \text{ and } \mathcal{E}(\vec{c}[2]) = (\vec{a}[1]@10 \oplus k_1) \oplus (\vec{a}[1]@11 \oplus k_2). \quad \square$$

**Discussion on the Program Syntax.** For convenience, our verification tool supports bounded for-loops (e.g., the for-loop in the gadget XOR) which are automatically and fully unfolded before verification, thus we only present the core language without loops. One may notice that the program syntax has a specific format, where (1) inputs and outputs of both simple and composite gadgets are vectors, (2) encoding and main gadgets take only scalar variables as inputs and return a vector, and (3) both composite and main gadgets only consist of call statements except for the return statement. This specific format is consistent with the design pattern of masked implementations of cryptographic algorithms and is widely adopted in the literature [9–11, 14, 15, 18, 19, 30, 31, 63, 87], thus our approach has a wide application on masked implementations of cryptographic algorithms. In a nutshell, the design process of a masked implementation starts by choosing a masking scheme (e.g., Boolean masking) that is implemented by an encoding gadget, thus an encoding gadget takes a scalar variable as input and produces an encoding of the scalar variable. Then, one will design a masked version for each operation used in the cryptographic algorithm, and the masked version performs the desired computation on the encodings of the operands of the operation. For the sake of code reuse, modularity, and maintenance in software development, masked versions of all the operations of cryptographic algorithms are wrapped as gadgets (i.e., functions), thus the inputs and output of a simple or composite gadget are encodings that correspond to the operands and results of the operation, respectively. Finally, each original function in the cryptographic algorithm becomes a composite gadget, where the inputs and output are replaced by the corresponding encodings, and each statement is replaced by a gadget call statement to the corresponding gadget. Similarly, the main function is revised by adding encoding call statements and gadget call statements, resulting in a main gadget.

**Semantics.** Given a variable set  $V$ , we denote by  $\eta : V \rightarrow \mathbb{F}$  the valuation of  $V$ , which maps variables to concrete values, and denote by  $\mathcal{D}(V)$  the set of all the joint distributions of  $V$ .

Let us fix a program  $P$ . Given a valuation  $\eta$  of the inputs  $X_k \cup X_p$  of the program  $P$ , the computation  $\mathcal{E}(x)$  of a variable  $x$  in the program  $P_{in}$  is interpreted as a probability distribution, denoted by  $\llbracket \mathcal{E}(x) \rrbracket_\eta^P$ , where variables in  $\text{Var}(\mathcal{E}(x)) \cap (X_k \cup X_p)$  are instantiated by the valuation  $\eta$  and random variables are sampled uniformly. Given a subset of variables  $X$  of the program  $P_{in}$ , we also denote by  $\llbracket X \rrbracket_\eta^P$  the joint distribution of variables in  $X$  under the valuation  $\eta$ .

Given a gadget  $g$ , let  $\mu \in \mathcal{D}(X_a^g)$  be a joint distribution of the formal parameters  $X_a^g$ . For each  $x \in X^g \cup X_a^g$ , the computation  $\mathcal{E}(x)$  is interpreted as a probability distribution, denoted by  $\llbracket \mathcal{E}(x) \rrbracket_\mu^g$ , where variables in  $\text{Var}(\mathcal{E}(x)) \cap X_a^g$  are sampled from  $\mu$  and  $\text{Var}(\mathcal{E}(x)) \cap X_r^g$  are sampled uniformly.

*Example 2.2.* Consider the variables  $\vec{c}[1]$  and  $\vec{c}[2]$  in the example  $P$  shown in Figure 4. Recall that

$$\mathcal{E}(\vec{c}[1]) = \vec{a}[1]@10 \oplus \vec{a}[1]@11 \text{ and } \mathcal{E}(\vec{c}[2]) = (\vec{a}[1]@10 \oplus k_1) \oplus (\vec{a}[1]@11 \oplus k_2).$$

Then, for any valuation  $\eta$  of the inputs  $\{k_1, k_2\}$ , the probability distributions  $\llbracket \mathcal{E}(\vec{c}[1]) \rrbracket_\eta^P$ ,  $\llbracket \mathcal{E}(\vec{c}[2]) \rrbracket_\eta^P$  and  $\llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P$  are defined as follows:

- $\llbracket \mathcal{E}(\vec{c}[1]) \rrbracket_\eta^P(0) = \llbracket \mathcal{E}(\vec{c}[1]) \rrbracket_\eta^P(1) = \llbracket \mathcal{E}(\vec{c}[2]) \rrbracket_\eta^P(0) = \llbracket \mathcal{E}(\vec{c}[2]) \rrbracket_\eta^P(1) = \frac{1}{2}$ , because  $\vec{a}[1]@10$  and  $\vec{a}[1]@11$  are random variables.
- If  $\eta(k_1) \oplus \eta(k_2) = 0$ :  $\llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P(0, 0) = \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P(1, 1) = \frac{1}{2}$  and  $\llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P(0, 1) = \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P(1, 0) = 0$ , because the random variables  $\vec{a}[1]@10$  and  $\vec{a}[1]@11$  are used in both  $\mathcal{E}(\vec{c}[1])$  and  $\mathcal{E}(\vec{c}[2])$ , causing interference between the values of  $\mathcal{E}(\vec{c}[1])$  and  $\mathcal{E}(\vec{c}[2])$ .
- If  $\eta(k_1) \oplus \eta(k_2) = 1$ :  $\llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P(0, 1) = \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P(1, 0) = \frac{1}{2}$  and  $\llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P(0, 0) = \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P(1, 0) = 0$ .  $\square$

## 2.2 Thread Model, Leakage Model, and Security Notions

**Thread Model and Leakage Model.** In this work, we adopt the widely used threat model [47, 48, 53, 76, 77, 81, 98], where the adversary knows the details of the implementation and has the ability to choose the values for the public input variables  $X_p$ , but does not know the values of the private input variables  $X_k$ , of the program  $P$ . Moreover, the adversary may have access to the results of one arbitrary-chosen intermediate computation (i.e., observable variable in  $P_{in}$ ) via power side-channel information, where all variables in the program  $P_{in}$  except for the private input variables  $X_k$  are observable variables to the adversary. Under these assumptions, the adversary's goal is to infer the value of private inputs  $X_k$ .

The power side-channel attacks exploit the correlation between power consumption values rather than the absolute power consumption. In this work, we consider the correlation between power consumption values that comes from the static leakage currents of CMOS transistors, where power consumption volume depends on whether the transistor is on or off, corresponding to the logical 1 and 0 of a bit, respectively. Thus, the value of a variable is proportional to the power consumption of storing the value. For example, if the device executes the statement  $a = k \oplus p$ , where  $k$  is the secret and  $p$  is the public, assuming that  $k$  and  $p$  are Boolean variables and  $p = 1$ , then  $a = 1$  if  $k = 0$  and  $a = 0$  if  $k = 1$ . The value of  $a$  thus depends on the value of  $k$ . The value of  $a$ , hence the value of  $k$ , can be deduced by analyzing the power consumption of executing the statement  $a = k \oplus p$ .

**Simulatability.** To characterize that a set of variables is statistically independent upon another variable set, we introduce the notions of randomized function and simulatability.

A *randomized function*  $\pi : \mathbb{F}^{|I|} \rightarrow \mathbb{F}^{|O|}$  over two sets of variables  $I$  and  $O$  is a function such that for any fixed tuple of values  $(v_1, \dots, v_{|I|}) \in \mathbb{F}^{|I|}$ ,  $\pi(v_1, \dots, v_{|I|})$  is a (joint) distribution of the values of the variables in  $O$ .

Given a set of variables  $I$ , a set of variables  $O$  can be simulated by the set of variables  $I$ , called *I-simulatable*, if there exists a randomized function  $\pi : \mathbb{F}^{|I|} \rightarrow \mathbb{F}^{|O|}$  such that for any fixed tuple of values  $(v_1, \dots, v_{|I|}) \in \mathbb{F}^{|I|}$  and any valuation  $\eta : X_a^{\text{main}} \rightarrow \mathbb{F}$ , the (joint) distributions  $\pi(v_1, \dots, v_{|I|})$  and  $\llbracket O \rrbracket_\eta^P$  are the same when the values of  $I$  in the program  $P$  are limited to  $(v_1, \dots, v_{|I|})$ .

Intuitively, when the values of variables in the set  $I$  are fixed, the joint distribution of (the values of) the variables in set  $O$  is fixed as well. In other words, knowing the values of the variables in  $I$  suffices to simulate the distribution  $\llbracket O \rrbracket_\eta^P$ . For example, consider  $y = x \oplus r$  and  $y' = x \wedge r$ , where  $x$  is an input variable and  $r$  is a random variable. The variable  $y$  is  $\emptyset$ -simulatable, meaning that the distribution of  $y$  can be simulated without knowing the values of any variables. However,  $y'$  is  $\{x\}$ -simulatable instead of  $\emptyset$ -simulatable, meaning that the distribution of  $y$  can be simulated when knowing the value of  $x$ .

Similarly, for a gadget  $g$  and a variable set  $I$ , a variable set  $O$  is *I-simulatable* if there exists a randomized function  $\pi : \mathbb{F}^{|I|} \rightarrow \mathbb{F}^{|O|}$  such that for any fixed tuple of values  $(v_1, \dots, v_{|I|}) \in \mathbb{F}^{|I|}$  and any distribution  $\mu \in \mathcal{D}(X_a^g)$ , the distributions  $\pi(v_1, \dots, v_{|I|})$  and  $\llbracket O \rrbracket_\mu^g$  are the same when the values of  $I$  in the gadget  $g$  are limited to  $(v_1, \dots, v_{|I|})$ .

It is straightforward to obtain the following proposition.

**PROPOSITION 2.3.** *Suppose the variable set  $O$  is I-simulatable. The following statements hold:*

- $O$  is  $I'$ -simulatable for any  $I'$  such that  $I \subseteq I'$ .
- $O'$  is  $I$ -simulatable for any  $O'$  such that  $O' \subseteq O$  or  $O'$  is  $O$ -simulatable. □

According to Proposition 2.3, for any variable set  $O$ , there always exists a variable set  $I$  (e.g.,  $O$ ) such that  $O$  is  $I$ -simulatable. We will see later that the smaller the size of  $I$ , the better for proving security. In Section 5.1, we will present a sound proof system for checking simulatability.

Hereafter, for the sake of presentation, a singleton set  $\{x\}$  may be directly written as  $x$ —for example,  $\{y\}$  is  $\{x\}$ -simulatable is written as  $y$  is  $x$ -simulatable.

**Probing Security.** A program  $P$  is *first-order probing secure* if each observable variable  $x$  in the program  $P_{in}$  is  $X_p$ -simulatable. Intuitively, the program  $P$  is first-order probing secure if the distribution of each observable variable  $x$  can be simulated by only knowing the values of public inputs. Thus, the distribution of the observable variable  $x$  is (statistically) independent of the private inputs  $X_k$  and the adversary cannot infer any information of the private inputs  $X_k$  when he/she has access to the value of the observable variable  $x$  via power side channels. The first-order probing security exactly characterizes the leakage model under the thread model introduced previously.

**Example 2.4.** Consider the program  $P$  shown in Figure 4. For any observable variable  $x \in X^{\text{main}_{in}}$ , we can examine that  $\llbracket \mathcal{E}(x) \rrbracket_\eta^P = \frac{1}{2}$  for any valuation  $\eta$  of the inputs  $\{k_1, k_2\}$ . It implies that  $x$  is  $\emptyset$ -simulatable, thus the program  $P$  is first-order probing secure.

In contrast,  $\llbracket \vec{c}[1], \vec{c}[2] \rrbracket_{\eta_1}^P \neq \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_{\eta_2}^P$  if  $\eta_1(k_1) \oplus \eta_1(k_2) = 0$  and  $\eta_2(k_1) \oplus \eta_2(k_2) = 1$ . Indeed, if  $\eta_1(k_1) \oplus \eta_1(k_2) = 0$  and  $\eta_2(k_1) \oplus \eta_2(k_2) = 1$ , then

$$\begin{aligned} & - \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_{\eta_1}^P(0, 0) = \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_{\eta_1}^P(1, 1) = \frac{1}{2}, \text{ and} \\ & - \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_{\eta_2}^P(0, 1) = \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_{\eta_2}^P(1, 0) = \frac{1}{2}. \end{aligned}$$

Thus,  $\{\vec{c}[1], \vec{c}[2]\}$  is not  $\emptyset$ -simulatable. By fixing the inputs  $\{k_1, k_2\}$ ,  $\llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P = \llbracket \vec{c}[1], \vec{c}[2] \rrbracket_\eta^P$ , thus  $\{\vec{c}[1], \vec{c}[2]\}$  becomes  $\{k_1, k_2\}$ -simulatable. □

```

1  main( $k_1, k_2$ ){
2     $\vec{a}$  = Encoding( $k_1$ );
3     $\vec{b}$  = Encoding( $k_2$ );
4     $\vec{c}$  = XORMULTI( $\vec{a}, \vec{b}$ );}
5    //  $\bigoplus \vec{c} = k_1 \odot (k_1 \oplus k_2)$ 
6
7  XORMULTI( $\vec{a}, \vec{b}$ ){
8     $\vec{e}$  = Refresh( $\vec{a}$ );
9     $\vec{c}$  = XOR( $\vec{b}, \vec{e}$ );
10    $\vec{d}$  = UMA( $\vec{e}, \vec{c}$ );
11   return  $\vec{d}$ ;}
12 //  $\bigoplus \vec{d} = (\bigoplus \vec{a}) \odot (\bigoplus \vec{a} \oplus \bigoplus \vec{b})$ 

13 Encoding( $k$ ){
14    $\vec{a}[1] = \$$ ;
15    $\vec{a}[2] = \vec{a}[1] \oplus k$ ;
16   return  $\vec{a}$ ;} //  $\bigoplus \vec{a} = k$ 
17 Refresh( $\vec{a}$ ){
18    $r_1 = \$$ ;
19    $\vec{c}[1] = \vec{a}[1] \oplus r_1$ ;
20    $\vec{c}[2] = \vec{a}[2] \oplus r_1$ ;
21   return  $\vec{c}$ ;} //  $\bigoplus \vec{c} = \bigoplus \vec{a}$ 
22 XOR( $\vec{a}, \vec{b}$ ){
23   for( $i = 1; i \leq 2; i++$ ){
24      $\vec{c}[i] = \vec{a}[i] \oplus \vec{b}[i]$ ;}
25   return  $\vec{c}$ ;} //  $\bigoplus \vec{c} = \bigoplus \vec{a} \oplus \bigoplus \vec{b}$ 

26 UMA( $\vec{a}, \vec{b}$ ){ // Unified Multiplication
27    $t_1 = \vec{a}[1] \odot \vec{b}[1]$ ;    $t_2 = \vec{a}[2] \odot \vec{b}[2]$ ;    $t_3 = \vec{a}[1] \odot \vec{b}[2]$ ;
28    $t_4 = \vec{a}[2] \odot \vec{b}[1]$ ;    $r_2 = \$$ ;
29    $t_5 = t_1 \oplus r_2$ ;       //  $t_5 = (\vec{a}[1] \odot \vec{b}[1]) \oplus r_2$ 
30    $t_6 = t_5 \oplus t_3$ ;     //  $t_6 = (\vec{a}[1] \odot \vec{b}[1]) \oplus r_2 \oplus (\vec{a}[1] \odot \vec{b}[2])$ 
31    $t_7 = t_2 \oplus r_2$ ;     //  $t_7 = (\vec{a}[2] \odot \vec{b}[2]) \oplus r_2$ 
32    $t_8 = t_7 \oplus t_4$ ;     //  $t_8 = (\vec{a}[2] \odot \vec{b}[2]) \oplus r_2 \oplus (\vec{a}[2] \odot \vec{b}[1])$ 
33   return ( $t_6, t_8$ );} //  $t_6 \oplus t_8 = (\bigoplus \vec{a}) \odot (\bigoplus \vec{b})$ 

```

Fig. 5. A running example for computing  $k_1 \odot (k_1 \oplus k_2)$ .

### 2.3 A Running Example

Suppose one wants to compute  $k_1 \odot (k_1 \oplus k_2)$ , where  $k_1$  and  $k_2$  are two private inputs. It is known that the power consumption of computing both  $k_1 \oplus k_2$  and  $k_1 \odot (k_1 \oplus k_2)$  depends on the values of  $k_1$  and  $k_2$ , based on which *differential power analysis* [72] could be utilized to infer the information of  $k_1$  and  $k_2$ . For instance,  $k_1 \oplus k_2$  is always 0 if  $k_1 = k_2$  and is always 1 if  $k_1 \neq k_2$ . The static leakage current of a CMOS transistor depends on whether the transistor is on or off, which corresponds to the logical 1 and 0 of a bit. This difference allows the adversary to infer if  $k_1 = k_2$  or not by measuring the power consumption.

To ensure first-order probing security of computing  $k_1 \odot (k_1 \oplus k_2)$ , the distributions of internal variables for computing  $k_1 \oplus k_2$  and  $k_1 \odot (k_1 \oplus k_2)$  should be (statistically) independent of the values of  $k_1$  and  $k_2$ . Namely, the distribution of static leakage currents of each CMOS transistor is independent of the values of  $k_1$  and  $k_2$ .

Figure 5 shows a demonstrating masked implementation for computing  $k_1 \odot (k_1 \oplus k_2)$  using Boolean masking. Given the inputs  $k_1$  and  $k_2$ , their encodings  $\vec{a}$  and  $\vec{b}$  are computed by calling the encoding gadget Encoding with the inputs  $k_1$  and  $k_2$ , respectively (lines 2 and 3). The Encoding gadget splits an input  $k$  into two shares  $\vec{a}[1]$  and  $\vec{a}[2]$ , where  $\vec{a}[1]$  is randomly sampled and  $\vec{a}[2] = \vec{a}[1] \oplus k$  masks  $k$  by XORing  $\vec{a}[1]$  resulting in the encoding  $\vec{a}$  of  $k$ .

Based on the encodings  $\vec{a}$  and  $\vec{b}$ , the encoding  $\vec{c}$  of  $k_1 \odot (k_1 \oplus k_2)$  is computed via invoking the composite gadget XORMULTI (line 4). The value of  $k_1 \odot (k_1 \oplus k_2)$  can be recovered by demasking (i.e.,  $\bigoplus \vec{c} = \vec{c}[1] \oplus \vec{c}[2] = k_1 \odot (k_1 \oplus k_2)$ ). Three simple gadgets Refresh [63], XOR [63], and UMA [58] are invoked in the gadget XORMULTI. The gadget Refresh takes an encoding as input and re-masks the shares via a fresh random variable. Refresh is required when the same random variable is used into two different sub-expressions in the same computation (e.g.,  $\vec{a}[2] \odot \vec{a}[1] = (\vec{a}[1] \oplus k_1) \odot \vec{a}[1]$  depends on the private variable  $k_1$ ). The gadget XOR performs share-wise XOR—that is, it produces the

encoding  $(\vec{a}[1] \oplus \vec{b}[1], \vec{a}[2] \oplus \vec{b}[2])$  for the input encodings  $\vec{a}$  and  $\vec{b}$ . The gadget UMA implements the finite-filed multiplication of the input encodings  $\vec{a}$  and  $\vec{b}$ , so the output encoding  $(t_6, t_8)$  satisfying  $t_6 \oplus t_8 = (\bigoplus \vec{a}) \odot (\bigoplus \vec{b})$ .

Consider the variable  $\vec{c}[1]@8@4$  which is the inlined version of the local variable  $\vec{c}[1]$  in the gadget Refresh after inlining the gadget call to Refresh at line 8 and the gadget call to the gadget XORMULTI at line 4, respectively. Obviously,  $\mathcal{E}(\vec{c}[1]@8@4) = (\vec{a}[1]@2 \oplus k_1) \oplus r_1@8@4$ . Since  $k_1$  in  $\mathcal{E}(\vec{c}[1]@8@4)$  is perfectly masked by the random variable  $r_1@8@4$  (respectively,  $\vec{a}[1]@2$ ), we can get that  $\llbracket \mathcal{E}(\vec{c}[1]@8@4) \rrbracket_{\eta_1}^P = \llbracket \mathcal{E}(\vec{c}[1]@8@4) \rrbracket_{\eta_2}^P$  for any valuations  $\eta_1, \eta_2 : \{k_1, k_2\} \rightarrow \mathbb{F}$ . Thus,  $\vec{c}[1]@8@4$  is  $\emptyset$ -simulatable. Indeed, we can observe that the local variable  $\vec{c}[1]$  in the simple gadget Refresh is perfectly masked by the random variable  $r_1$  without considering the calling context. This means that  $\vec{c}[1]$  is always independent of private inputs no matter how the gadget Refresh is invoked. Similarly, consider the local variable  $t_5$  in the gadget UMA. Since  $t_5 = (\vec{a}[1] \odot \vec{b}[1]) \oplus r_2$ , we can deduce that  $t_5$  is always independent of private inputs due to the random variable  $r_2$  and operation  $\oplus$  no matter how the gadget UMA is invoked.

**Outline of the Solution.** To automatically prove that masked implementations of cryptographic algorithms such as the running example are first-order probing secure without inlining gadget calls, we shall propose a compositional verification approach by leveraging *local* random variables. We first introduce the new security notion  $\mathbb{I}$ -Non-Interference in terms of simulatability, which is parameterized by an explicit pre-condition  $\mathbb{I}$  for each gadget (Section 3.1). The pre-condition of each gadget is served as an abstraction of the gadget when reasoning about the calls to the gadget so that the inlining of the gadget calls can be avoided. Then, we propose a composition rule for inferring pre-condition  $\mathbb{I}$  of composite gadgets using the pre-conditions of called gadgets without inlining gadget calls (Section 3.2). The new security notion and its composition rule lay the foundation of our compositional verification approach. To reduce verification time, we further propose the concepts of dominant and dominated variables (Section 4.1), present a sound approach for determining them (Section 4.2), and show how to use them to reduce the size of pre-condition  $\mathbb{I}$  for a composite gadget by revising the composition rule. Finally, after introducing a proof system for checking simulatability (Section 5.1), we present algorithms to infer pre-conditions for simple gadgets (Section 5.2) and composite gadgets (Section 5.3), respectively, by utilizing the proposed proof system and composition rules.

### 3 FOUNDATION OF OUR COMPOSITIONAL VERIFICATION APPROACH

In this section, we first introduce the new security notion  $\mathbb{I}$ -Non-Interference in terms of simulatability, parameterized by an explicit pre-condition  $\mathbb{I}$  for each gadget, and then discuss how to make composition based on  $\mathbb{I}$ -Non-Interference by presenting a compositional rule.

#### 3.1 $\mathbb{I}$ -Non-Interference

To achieve compositional verification for masked implementations of cryptographic algorithms, we propose the notion of  $\mathbb{I}$ -Non-Interference. Let  $\mathcal{P}(\cdot)$  denote the power set of a set.

*Definition 3.1.* Given a set of variable sets  $\mathbb{I} \subseteq \mathcal{P}(X^{g_{in}} \cup X_a^g)$  of a gadget  $g$ , the gadget  $g$  is  $\mathbb{I}$ -Non-Interfering ( $\mathbb{I}$ -NI for short), if for every variable  $x \in X^{g_{in}} \cup X_a^g$ , there exists a variable set  $I \in \mathbb{I}$  such that  $x$  is  $I$ -simulatable. (Note that  $g_{in}$  is the inlined version of the gadget  $g$  and  $X^{g_{in}}$  is the set of internal variables defined in the inlined version  $g_{in}$ .)

The pre-condition  $\mathbb{I}$  in the notion  $\mathbb{I}$ -NI characterizes sufficient variable sets to simulate the distributions of all observable variables. The advantage of the pre-condition  $\mathbb{I}$  is twofold. First, it can contain as few small variable sets as possible to simulate all the observable variables in a gadget.



Second, if an observable variable  $x$  cannot be simulated by any variable set into  $\mathbb{I}$ , the variable  $x$  could be added to the set  $\mathbb{I}$  to ensure that the gadget is  $\mathbb{I}$ -NI. The new notion  $\mathbb{I}$ -NI can be seen as a balance of the first-order probing security and the stronger security notion proposed by Barthe et al. [9], where the former does not support compositional verification, whereas the latter implicitly contains a fixed pre-condition. We note that the fixed pre-condition may not be fulfilled by generic, in particular, efficient, masked implementations, although it is more composition-friendly. More importantly, it is straightforward to check first-order probing security from our notion  $\mathbb{I}$ -NI by the following proposition.

**PROPOSITION 3.2.** *If a gadget  $g$  is both  $\mathbb{I}$ -NI and  $I$  is  $X_p$ -simulatable for each variable set  $I \in \mathbb{I}$ , then  $g$  is first-order probing secure, namely any variable of the gadget  $g$  is statistically independent of the private inputs  $X_k$  when  $g$  is used in a program  $P$  with the private inputs  $X_k$ .*

**PROOF.** Suppose the gadget  $g$  is both  $\mathbb{I}$ -NI and  $I$  is  $X_p$ -simulatable for each variable set  $I \in \mathbb{I}$ . To show that  $g$  is first-order probing secure, it suffices to prove that for any program  $P$  that uses the gadget  $g$  and any observable variable  $x \in X^{gin} \cup X_a^g$ ,  $x@$  is  $X_p$ -simulatable (i.e., any inlined version  $x@$  of  $x$  in the program  $P_{in}$  is statistically independent of the private inputs  $X_k$ ).

Consider an observable variable  $x \in X^{gin} \cup X_a^g$ . Since the gadget  $g$  is  $\mathbb{I}$ -NI, there must exist a variable set  $I \in \mathbb{I}$  such that  $x$  is  $I$ -simulatable. By Proposition 2.3, we get that for any inlined  $x@$  version of the variable  $x$  from the gadget  $g$  in the program  $P_{in}$ ,  $x@$  is  $X_p$ -simulatable.  $\square$

**Example 3.3.** Consider the running example. Let  $\mathbb{I}_{Refresh}$ ,  $\mathbb{I}_{XOR}$  and  $\mathbb{I}_{UMA}$  be the following sets:

- $\mathbb{I}_{Refresh} = \{\{\vec{a}[1]\}, \{\vec{a}[2]\}\}$ ;
- $\mathbb{I}_{XOR} = \{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}\}$ ;
- $\mathbb{I}_{UMA} = \{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}, \{\vec{a}[1], \vec{b}[2]\}, \{\vec{a}[2], \vec{b}[1]\}\}$ .

It is easy to verify the following:

- In Refresh,  $r_1$  and  $\vec{c}[1]$  and  $\vec{c}[2]$  are  $\emptyset$ -simulatable, and  $\vec{a}[i]$  is  $\vec{a}[i]$ -simulatable for  $i = 1, 2$ .
- In XOR,  $\vec{a}[i]$  is  $\vec{a}[i]$ -simulatable,  $\vec{b}[i]$  is  $\vec{b}[i]$ -simulatable, and  $\vec{c}[i]$  is  $\{\vec{a}[i], \vec{b}[i]\}$ -simulatable for  $i = 1, 2$ .
- In UMA,  $\vec{a}[i]$  is  $\vec{a}[i]$ -simulatable,  $\vec{b}[i]$  is  $\vec{b}[i]$ -simulatable,  $\vec{t}[i]$  is  $\{\vec{a}[i], \vec{b}[i]\}$ -simulatable for  $i = 1, 2$ ,  $t_3$  is  $\{\vec{a}[1], \vec{b}[2]\}$ -simulatable,  $t_4$  is  $\{\vec{a}[2], \vec{b}[1]\}$ -simulatable, and all of  $r_2, t_5, t_6, t_7$ , and  $t_8$  are  $\emptyset$ -simulatable.

Thus, Refresh is  $\mathbb{I}_{Refresh}$ -NI, XOR is  $\mathbb{I}_{XOR}$ -NI, and UMA is  $\mathbb{I}_{UMA}$ -NI.  $\square$

In Section 5.2, we will present an algorithm for inferring pre-conditions of simple gadgets. An algorithm for inferring pre-conditions of composite gadgets will be given in Section 5.3, which relies upon pre-conditions of simple gadgets and the composition rule introduced next.

### 3.2 Composition of $\mathbb{I}$ -NI Gadgets

We show the simplicity of inferring pre-conditions for composite gadgets by proposing a composition rule. We use the following composite gadget to illustrate the composition rule,

$$f(\vec{x}_1, \dots, \vec{x}_m) \{ \vec{y}_1 = g(\vec{x}_1, \dots, \vec{x}_m); \vec{z} = h(\vec{y}_1, \vec{x}_2, \dots, \vec{x}_n); \text{return } \vec{z}; \},$$

where  $f$  is a composite gadget that contains two gadget call statements to the simple gadgets  $g$  and  $h$  (with labels  $\ell_g$  and  $\ell_h$ ), respectively. The actual parameters of  $g$  are the formal parameters of  $f$ , and the actual parameters of  $h$  consist of the return values of  $g$  and formal parameters of  $f$ . Assume the gadget  $g(\vec{a}_1, \dots, \vec{a}_m)\{\dots\}$  is  $\mathbb{I}_g$ -NI and the gadget  $h(\vec{b}_1, \dots, \vec{b}_n)\{\dots\}$  is  $\mathbb{I}_h$ -NI. We denote by  $\mathbb{I}_g[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@_{\ell_g}$  the set  $\mathbb{I}_g$  after instantiating the formal parameters  $\vec{a}_i$ 's by the

corresponding actual parameters  $\vec{x}_i$ 's, and local variables are appended with  $@\ell_g$  to avoid name conflict. The set  $\mathbb{I}_h[\vec{y}_1/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@\ell_h$  is defined similarly. Let  $\mathbb{I}_f$  be the following set:

$$\mathbb{I}_g[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@\ell_g \cup \mathbb{I}_h[\vec{y}_1/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@\ell_h.$$

PROPOSITION 3.4. *The gadget  $f$  is  $\mathbb{I}_f$ -NI.*

PROOF. To prove that the gadget  $f$  is  $\mathbb{I}_f$ -NI, it suffices to prove that for every observable variable  $x$  of the inlined version  $f_{in}$ , the variable  $x$  in  $f_{in}$  is  $I$ -simulatable for some variable set  $I \in \mathbb{I}_f$ . We first consider inlined versions of the variables from the gadgets  $h$  and  $f$ , then move on to the observable variables defined in the gadget  $f$ :

- Consider an inlined version  $x@\ell_g$  of a variable  $x$  from the gadget  $g$ . Since the gadget  $g$  is  $\mathbb{I}_g$ -NI, there exists a variable set  $I \in \mathbb{I}_g$  such that the variable  $x$  in  $g$  is  $I$ -simulatable. It implies that the variable  $x@\ell_g$  in  $f_{in}$  is  $I[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@\ell_g$ -simulatable. The result immediately follows from the fact that  $I[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@\ell_g \in \mathbb{I}_g[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@\ell_g \subseteq \mathbb{I}_f$ .
- Considering an inlined version  $x@\ell_h$  of a variable  $x$  from the gadget  $h$ , similar to the preceding case, we can get that the variable  $x@\ell_h$  in  $f_{in}$  is  $I[\vec{y}_1/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@\ell_h$ -simulatable for some set  $I \in \mathbb{I}_h$  and  $I[\vec{y}_1/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@\ell_h \in \mathbb{I}_h[\vec{y}_1/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@\ell_h \subseteq \mathbb{I}_f$ .
- Consider an observable variable  $x$  defined in the gadget  $f$ . Then,  $x$  must be an actual parameter or a return value of one of the gadget call statements  $\vec{y}_1 = g(\vec{x}_1, \dots, \vec{x}_m)$  or  $\vec{z} = h(\vec{y}_1, \vec{x}_2, \dots, \vec{x}_n)$ .

If  $x$  is an actual parameter of  $\vec{y}_1 = g(\vec{x}_1, \dots, \vec{x}_m)$  or  $\vec{z} = h(\vec{y}_1, \vec{x}_2, \dots, \vec{x}_n)$ , let  $c$  be the corresponding formal parameter of  $x$ . Then, the variable  $c$  in the gadget  $g$  is  $I$ -simulatable for some  $I \in \mathbb{I}_g$  or the variable  $c$  in the gadget  $h$  is  $I$ -simulatable for some  $I \in \mathbb{I}_h$ . Since  $x$  and  $c$  always have the same value, we get that the variable  $x$  in  $f_{in}$  is  $I'$ -simulatable, where  $I'$  is  $I[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@\ell_g$  or  $I[\vec{y}_1/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@\ell_h$ .

If  $x$  is a return value of  $\vec{y}_1 = g(\vec{x}_1, \dots, \vec{x}_m)$  or  $\vec{z} = h(\vec{y}_1, \vec{x}_2, \dots, \vec{x}_n)$ , let  $c$  be the corresponding return variable of  $x$  in the gadget  $g$  or  $h$ . We have proved that  $c@\ell_g$  (respectively,  $c@\ell_h$ ) is  $I$ -simulatable if  $c$  is a return variable of  $g$  (respectively,  $h$ ) for some  $I \in \mathbb{I}_f$ . Since  $x$  and  $c$  always have the same value, we get that the variable  $x$  in the gadget  $f_{in}$  is  $I$ -simulatable as well.

We have proved that every observable variable  $x$  of the inlined version  $f_{in}$  is  $I$ -simulatable, for some variable set  $I \in \mathbb{I}_f$ , and thus conclude the proof.  $\square$

*Example 3.5.* Here we show how to manually compute the pre-condition  $\mathbb{I}_{\text{XORMULTI}}$  such that the composite gadget `XORMULTI` in the running example is  $\mathbb{I}_{\text{XORMULTI}}$ -NI by leveraging this composition rule. (An algorithm for inferring pre-conditions of composite gadgets will be given in Section 5.3.) Recall that `Refresh` is  $\mathbb{I}_{\text{Refresh}}$ -NI, the gadget `XOR` is  $\mathbb{I}_{\text{XOR}}$ -NI, and the gadget `UMA` is  $\mathbb{I}_{\text{UMA}}$ -NI (cf. Example 3.3), where

- $\mathbb{I}_{\text{Refresh}} = \{\{\vec{a}[1]\}, \{\vec{a}[2]\}\};$
- $\mathbb{I}_{\text{XOR}} = \{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}\};$  and
- $\mathbb{I}_{\text{UMA}} = \{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}, \{\vec{a}[1], \vec{b}[2]\}, \{\vec{a}[2], \vec{b}[1]\}\}.$

The set  $\mathbb{I}_{\text{XORMULTI}}$  is computed as follows according to the composition rule:

- (1) For the gadget call  $\vec{e} = \text{Refresh}(\vec{a})$ , we have

$$\mathbb{I}_{\text{Refresh}}@\delta = \mathbb{I}_{\text{Refresh}}[\vec{a}/\vec{a}]@\delta = \{\{\vec{a}[1]\}, \{\vec{a}[2]\}\}.$$

(2) For the gadget call  $\vec{c} = \text{XOR}(\vec{b}, \vec{e})$ , we have

$$\mathbb{I}_{\text{XOR}@9} = \mathbb{I}_{\text{XOR}}[\vec{b}/\vec{a}, \vec{e}/\vec{b}]@9 = \{\{\vec{b}[1], \vec{e}[1]\}, \{\vec{b}[2], \vec{e}[2]\}\}.$$

(3) For the gadget call  $\vec{d} = \text{UMA}(\vec{e}, \vec{c})$ , we have

$$\mathbb{I}_{\text{UMA}@10} = \mathbb{I}_{\text{UMA}}[\vec{e}/\vec{a}, \vec{c}/\vec{b}]@10 = \{\{\vec{e}[i], \vec{c}[j]\} | 1 \leq i, j \leq 2\}.$$

Let  $\mathbb{I}_{\text{XORMULTI}} = \mathbb{I}_{\text{Refresh}@8} \cup \mathbb{I}_{\text{XOR}@9} \cup \mathbb{I}_{\text{UMA}@10} = \{\{\vec{a}[i]\}, \{\vec{b}[i], \vec{e}[i]\}, \{\vec{e}[i], \vec{c}[j]\} | 1 \leq i, j \leq 2\}$ . By Proposition 3.4, XORMULTI is  $\mathbb{I}_{\text{XORMULTI}}$ -NI.  $\square$

## 4 DOMINANT AND DOMINATED VARIABLES

As mentioned previously, the pre-condition  $\mathbb{I}$  of a gadget is expected to contain as few small variable sets as possible to simulate all observable variables. However, directly applying the composition rule may yield a pre-condition containing many large variable sets. To alleviate this issue, in this section, we first introduce dominant and dominated variables, then provide an approach to identify them, and finally show how such information can help further reduce the size of pre-condition  $\mathbb{I}$  in the composition of gadgets by improving the composition rule.

### 4.1 The Concepts of Dominant and Dominated Variables

Given a variable  $x$ , let  $\text{Sub}(x)$  be the set of the sub-expressions of  $\mathcal{E}(x)$ .

*Definition 4.1.* A variable  $x$  is dominated by another variable  $y$  if  $y$  occurs in  $\mathcal{E}(x)$  only once and each operation  $\circ$  along the path from  $y$  to the root of the abstract syntax tree of  $\mathcal{E}(x)$  is either from or belongs to following:

- $\{\oplus, +, -, \neg\}$ ,
- $\odot$  and one of its children is non-zero constant,

where  $x$  is called a *dominated variable* and  $y$  is called a *dominant variable* of  $x$ .

An encoding  $\vec{x}$  is dominated by another encoding  $\vec{y}$  if each share  $\vec{x}[i]$  of the encoding  $\vec{x}$  is dominated by only one share  $\vec{y}[j]$  of the encoding  $\vec{y}$ , where  $\vec{x}$  is called *dominated encoding* and  $\vec{y}$  is called *dominant encoding* of  $\vec{x}$ .

Intuitively, if the variable  $x$  is dominated by the another variable  $y$ , the distribution of (the value of) of  $x$  is determined by the distribution of  $y$ . When  $y$  is a random variable, the distribution of  $x$  is uniform as well, thus  $x$  can be regarded as a random variable. For example, consider  $y = x \oplus r$ ,  $y' = x \wedge r$ , and  $y'' = x + (x \oplus r)$ . The variable  $y$  is dominated by both  $x$  and  $r$  because of the logical operation  $\oplus$  and the uniqueness of  $x$  and  $r$ , but  $y'$  is dominated by neither  $x$  nor  $r$  due to the logical operation  $\wedge$ , and  $y''$  is only dominated by  $r$  because  $x$  occurs twice. When  $r$  is a random variable, the distributions of  $y$  and  $y''$  are uniform for any fixed value of  $x$ .

Let  $\text{DomR}(x)$  be the set of random dominant variables of the variable  $x$ . The merit of dominated variables is justified by the following straightforward proposition.

**PROPOSITION 4.2.**  $x$  has a uniform distribution if  $\text{DomR}(x) \neq \emptyset$ .

**PROOF.** Suppose  $r \in \text{DomR}(x)$ . We first prove the following claim.

**Claim.** Let  $e$  be a sub-expression of  $\mathcal{E}(x)$  such that  $r$  occurs in  $e$  and  $\mathcal{E}(x)[r/e]$  be the expression  $\mathcal{E}(x)$  in which all the occurrences of the sub-expression  $e$  are replaced by the random variable  $r$ .  $\mathcal{E}(x)$  and  $\mathcal{E}(x)[r/e]$  have the same distribution if  $e$  is in the form of  $e' \circ r$  for  $\circ \in \{\oplus, +, -\}$  or  $\neg r$  or  $c \odot r$  such that  $c$  is non-zero constant.

*Proof of the Claim.* Since  $r \in \text{DomR}(x)$ , the random variable  $r$  occurs only once in  $\mathcal{E}(x)$ . It means that the random variable  $r$  occurs only once in  $e$  but does not appear in  $e'$ . Since  $r$  is uniformly sampled from  $\mathbb{F}$ , we get that the value of  $e$  is determined by the value of  $r$  for any fixed value of  $e'$  or  $c$ . Let  $f_v(r)$  be the function such that the value of  $e'$  or  $c$  is fixed to  $v$ . Obviously,  $f_v(r)$  is a bijective function. Thus, the probability distribution of  $f_v(r)$  is the same as  $r$ , implying that the probability distribution of the value of  $e$  is uniform when  $r$  is uniformly sampled from  $\mathbb{F}$ . Since  $r$  does not appear anywhere in  $\mathcal{E}(x)$  except for the sub-expression  $e$ , we get that  $\mathcal{E}(x)$  and  $\mathcal{E}(x)[r/e]$  have the same distribution. We conclude the proof of the claim.

To show that  $x$  has a uniform distribution, we can iteratively replace all the occurrences of the sub-expression  $e$  in  $\mathcal{E}(x)$  by the random variable  $r$  for every sub-expression  $e$  in the form of  $e' \circ r$  for  $\circ \in \{\oplus, +, -\}$  or  $-r$  or  $c \odot r$  such that  $c$  is non-zero constant. The expression  $\mathcal{E}(x)$  will eventually become the random variable  $r$  according to Definition 4.1. By the preceding lemma, these substitutions do not change the probability distribution of  $\mathcal{E}(x)$ . Thus,  $x$  has a uniform distribution.  $\square$

*Example 4.3.* Consider the gadget Refresh in the running example. Since  $\mathcal{E}(\vec{c}[1]) = \vec{a}[1] \oplus r_1$  and  $\mathcal{E}(\vec{c}[2]) = \vec{a}[2] \oplus r_1$ , we can deduce that  $\vec{c}[1]$  is a dominated variable and dominated by both  $\vec{a}[1]$  and  $r_1$ , and the variables  $\vec{a}[1]$  and  $r_1$  are dominant variables of  $\vec{c}[1]$ . Similarly,  $\vec{c}[2]$  is dominated by both  $\vec{a}[2]$  and  $r_1$ , and the variables  $\vec{a}[2]$  and  $r_1$  are dominant variables of  $\vec{c}[2]$ . Since  $r_1$  is a random variable, we get that  $\text{DomR}(\vec{c}[1]) = \text{DomR}(\vec{c}[2]) = \{r_1\}$ , and both  $\vec{c}[1]$  and  $\vec{c}[2]$  have a uniform distribution for any inputs of the gadget Refresh.

Since each share  $\vec{c}[i]$  of the encoding  $\vec{c}$  is dominated by only one share  $\vec{a}[i]$  of the encoding  $\vec{a}$ , we get that the encoding  $\vec{c}$  is a dominated encoding and dominated by the encoding  $\vec{a}$ , and the encoding  $\vec{a}$  is a dominant encoding of the encoding  $\vec{c}$ .

In contrast,  $\vec{c}[1]$  is dominated by neither  $\vec{a}[1]$  nor  $r_1$  if  $\mathcal{E}(\vec{c}[1])$  is  $\vec{a}[1] \wedge r_1$  (because of  $\wedge$ ) or  $(\vec{a}[1] \wedge r_1) \oplus r_1$  (because of twice occurrences of  $r_1$  and  $\wedge$ ). Thus,  $\text{DomR}(\vec{c}[1]) = \emptyset$  and  $\vec{c}[1]$  does not necessarily have a uniform distribution even though  $r_1$  is a random variable.

If  $\mathcal{E}(\vec{c}[1])$  is  $(\vec{a}[1] \vee \vec{a}[2]) \oplus (-r_1)$ ,  $\vec{c}[1]$  is only dominated by the variable  $r_1$  (because of  $\vee$ ), then the encoding  $\vec{c}$  is not dominated by the encoding  $\vec{a}$  because the share  $\vec{c}[1]$  is not dominated by any share of  $\vec{a}$ . Moreover,  $\text{DomR}(\vec{c}[1]) = \{r_1\}$  and  $\vec{c}[1]$  has a uniform distribution as  $r_1$  is a random variable.

If  $\mathcal{E}(\vec{c}[1])$  is  $(\vec{a}[1] \oplus \vec{a}[2]) \oplus r_1$ ,  $\vec{c}[1]$  is dominated by  $\vec{a}[1]$ ,  $\vec{a}[2]$ , and  $r_1$ , then the encoding  $\vec{c}$  is not dominated by the encoding  $\vec{a}$  because the share  $\vec{c}[1]$  of the encoding  $\vec{c}$  is dominated by both shares  $\vec{a}[1]$  and  $\vec{a}[2]$  of the encoding  $\vec{a}$ . Similarly,  $\text{DomR}(\vec{c}[1]) = \{r_1\}$  and  $\vec{c}[1]$  has a uniform distribution.  $\square$

Proposition 4.2 provides a sufficient condition to deduce that  $x$  has the uniform distribution and thus can be simulated by an empty set (i.e.,  $x$  is  $\emptyset$ -simulatable). We will also show that variables dominated by random variables can help simplify expressions later (cf. Section 5).

## 4.2 Determining Dominated Variables

In this section, we present an efficient approach to determine dominant and dominated variables. We first consider simple gadgets and then composite gadgets.

*4.2.1 Determining Dominated Variables for Simple Gadgets.* Determining dominated variables in simple gadgets is trivial. For every variable  $x$  of a simple gadget, we represent the computation  $\mathcal{E}(x)$  of the variable  $x$  as a **Directed Acyclic Graph (DAG)**, where the internal nodes are labeled by operators and leaves are labeled by variables and constants involved in the computation  $\mathcal{E}(x)$ . We iteratively traverse the DAG representation of  $\mathcal{E}(x)$  starting from the root in a depth-first fashion, where a stack is used to store visiting nodes. For each node in the DAG:

- (1) If the in-degree of the node is greater than 1, then we go back to its visiting parent (popped from the stack) if it exists and otherwise terminate, because any variable labeled to the leaf of the sub-tree rooted by this node occurs at least twice in the computation  $\mathcal{E}(x)$ .
- (2) If the in-degree of the node is 1, we proceed as follows:
  - (a) If the node is labeled by one of the operators  $\{\oplus, +, -, \neg\}$ , we push it into the stack and continue to traverse its unvisited children.
  - (b) If the node is labeled by the operator  $\odot$ , one child of the node is a non-zero constant, and the another child is a non-constant that has not been visited yet, we push it into the stack and continue to traverse its non-constant child.
  - (c) Otherwise, we go back to its visiting parent if it exists and otherwise terminate, because any variable labeled to the labeled to each leaf of sub-tree rooted by this node cannot be a dominant variable of  $x$  according to Definition 4.1.
- (3) If the node is a leaf, with in-degree no more than 1, and labeled by a variable, then the variable must be a dominant variable of  $x$ , and we record this dominant variable and go back to its visiting parent.

By applying the preceding procedure, we can identify all the dominant variables of the variable  $x$  in linear time in the size of the computation  $\mathcal{E}(x)$ . Similarly, the set  $\text{DomR}(x)$  of random dominant variables of the variable  $x$  can be computed.

**4.2.2 Determining Dominated Variables for Composite Gadgets.** It is non-trivial to determine if variables (encodings) are dominated by random variables in composite gadgets without inlining gadget calls, as they are return values of gadget calls. To address this issue, we first characterize whether a gadget can transfer dominant variables from a formal parameter to its output encoding and whether a gadget can generate dominant variables itself, based on which we introduce the concept of summary for storing this information. Next, we present an approach for under-approximating the summaries of composite gadgets by utilizing the summaries of called gadgets.

**Characterization of Gadgets.** The characterization of gadgets is inspired by following observation. First, the return of a gadget call statement may be dominated by the same random variables as some actual parameters of the gadget call statement—that is, random variables may be transferred from the input encodings to the output encoding, making the output encoding being dominated by the same dominant encodings of some input encodings. Second, the output encoding of a gadget may be dominated by local random variables of the gadget. Based on these observations, we characterize the gadgets that can transfer dominant variables and generate dominant variables as follows.

Fix a gadget  $g(\vec{a}_1, \dots, \vec{a}_m)\{\dots; \text{return } \vec{o}\}$ .

**Definition 4.4.** The gadget  $g$  can transfer dominant variables from a formal parameter  $\vec{a}_i$  to its output encoding  $\vec{o}$  if  $\vec{a}_i$  is a dominant encoding of  $\vec{o}$ .

**Definition 4.5.** The gadget  $g$  can generate dominant variables if each share  $\vec{o}[j]$  of the output encoding  $\vec{o}$  is dominated by a local random variable of  $g$ .

**Summary of Gadgets.** We define the summary of the gadget  $g$  as  $T_g \subseteq X_{en}^g \cup \{\vec{o}\}$ , where  $T_g$  includes all the formal parameters  $\vec{a}_i$  of the gadget  $g$  whose dominant encodings can be transferred to the output encoding  $\vec{o}$ , and the output encoding  $\vec{o}$  is added to the summary  $T_g$  if the gadget  $g$  itself can generate dominant variables. Based on Definitions 4.4 and 4.5, the summary  $T_g$  can be easily computed if  $g$  is a simple gadget. More specifically, for each share  $\vec{o}[j]$  of the output encoding  $\vec{o}$ , we first compute the dominant variables from the computation  $\mathcal{E}(\vec{o}[j])$ , then add an input encoding  $\vec{a}_i$  into the summary  $T_g$  if each share  $\vec{o}[j]$  of the output encoding  $\vec{o}$  is dominated by only one share



of the input encoding  $\vec{a}_i$ , and finally the output encoding  $\vec{o}$  is added into the summary  $T_g$  if each share  $\vec{o}[j]$  of the output encoding  $\vec{o}$  is dominated by a random variable.

*Example 4.6.* Consider the gadget Refresh in the running example. Since  $\vec{c}[1] = \vec{a}[1] \oplus r_1$  and  $\vec{c}[2] = \vec{a}[2] \oplus r_1$  with random variables  $r_1$  and  $r_2$ , Refresh can generate dominant variables (i.e.,  $r_1$  and  $r_2$ ), thus the output encoding  $\vec{c}$  of the gadget Refresh can be added into the summary  $T_{\text{Refresh}}$  of the gadget Refresh (i.e.,  $\vec{c} \in T_{\text{Refresh}}$ ). Since the output encoding  $\vec{c}$  of the gadget Refresh is dominated by the input encoding  $\vec{a}$ , the input encoding  $\vec{a}$  can also be added into the summary  $T_{\text{Refresh}}$  of the gadget Refresh (i.e.,  $\vec{a} \in T_{\text{Refresh}}$ ). Thus, we have  $T_{\text{Refresh}} = \{\vec{a}, \vec{c}\}$ . Similarly, we can deduce that  $T_{\text{XOR}} = \{\vec{a}, \vec{b}\}$  and  $T_{\text{UMA}} = \{(t_6, t_8)\}$ .  $\square$

**Under-Approximating the Summary of Composite Gadgets.** Let  $E$  be the set of all the encodings of the gadget  $g$ . To compute the summary  $T_g$  of the gadget  $g$  without inlining gadget calls when  $g$  is a composite gadget, we define a function  $\lambda : E \rightarrow \mathcal{P}(E)$  such that for each encoding  $\vec{a} \in E$ ,  $\lambda(\vec{a})$  contains all of the dominant encodings of the encoding  $\vec{a}$ , and moreover  $\vec{a} \in \lambda(\vec{a})$  only if  $\vec{a}$  is the return of a gadget call that can generate dominant variables. Thus, it is easy to see the following proposition.

**PROPOSITION 4.7.**  $\lambda(\vec{o}) \cap (X_{en}^g \cup \{\vec{o}\}) \subseteq T_g$ , namely the summary  $T_g$  of the composite gadget can be under-approximated by computing the function  $\lambda$ .

**PROOF.** According to the definition of the function  $\lambda$ , all the dominant encodings of the output encoding  $\vec{o}$  are contained in  $\lambda(\vec{o})$ . Thus, the intersection of  $\lambda(\vec{o})$  and  $X_{en}^g$  is a set of dominant encodings of the output encoding  $\vec{o}$  that are formal parameters of the gadget  $g$ . Furthermore,  $\vec{o} \in \lambda(\vec{o})$  only if  $\vec{o}$  is the return of a gadget call that can generate dominant variables, thus the gadget  $g$  itself can generate dominant variables. The result follows from the definition of  $T_g$ .  $\square$

Note that  $X_{en}^g$ , the vector of all the input encodings of the gadget  $g$ , is used as a set of input encodings in the definition of  $T_g$  and Proposition 4.7.

Now the problem is how to compute  $\lambda$ . To achieve this, we first show how to compute the dominant encodings  $\lambda(\vec{y})$  of an internal encoding  $\vec{y}$  defined by  $\vec{y} = f(\vec{x}_1, \dots, \vec{x}_k)$ , based on the summary  $T_f$  of the gadget  $f$  and dominant encodings of the actual parameters (i.e.,  $\lambda(\vec{x}_1), \dots, \lambda(\vec{x}_k)$ ). We assume that the formal parameters of the gadget  $f$  are  $\vec{b}_1, \dots, \vec{b}_k$ . First, we can directly get the following proposition.

**PROPOSITION 4.8.** *If the gadget  $f$  can generate dominant variables, then  $\vec{y}$  can be added into  $\lambda(\vec{y})$ .*

**PROOF.** Let  $\lambda(\vec{o})$  be the output encoding of the gadget  $f$ . Suppose the gadget  $f$  can generate dominant variables, then  $\vec{o} \in \lambda(\vec{o})$ . Since  $\vec{y} = f(\vec{x}_1, \dots, \vec{x}_k)$  (i.e.,  $\vec{y}$  is the return of a gadget call that can generate dominant variables), we can add  $\vec{y}$  into  $\lambda(\vec{y})$ .  $\square$

*Example 4.9.* Consider the gadget Refresh in the running example. It can generate dominant variables (i.e.,  $\vec{c} \in T_{\text{Refresh}}$ , cf. Example 4.6), thus  $\vec{c} \in \lambda(\vec{c})$ . Now, consider the gadget XORMULTI in the running example which has a gadget call to the gadget Refresh (i.e.,  $\vec{e} = \text{Refresh}(\vec{a})$ ). Thus, the encoding  $\vec{e}$  can be added into  $\lambda(\vec{e})$ .

In contrast, the gadget XOR in the running example cannot generate dominant variables (i.e.,  $\vec{c} \notin T_{\text{XOR}}$ , cf. Example 4.6). Thus, we cannot add the encoding  $\vec{c}$  into  $\lambda(\vec{c})$  for the gadget call statement  $\vec{c} = \text{XOR}(\vec{b}, \vec{e})$ .  $\square$

Hereafter, by mutually independence of the encodings  $\vec{x}_1, \dots, \vec{x}_k$ , we mean that the distributions of all the encodings  $\vec{x}_1, \dots, \vec{x}_k$  are mutually independent. Note that distributions of the shares in the same encoding can be dependent. For instance, the encodings  $\vec{a}$  and  $\vec{b}$  in the main gadget in

the running example are mutually independent, and the encodings  $\vec{b}$  and  $\vec{e}$  in the XORMULTI gadget in the running example are mutually independent as well, whereas the encodings  $(r, r \oplus k_1)$  and  $(r, r \oplus k_2)$  are not mutually independent even if  $r$  is a random variable. We have the following proposition.

**PROPOSITION 4.10.** *If the gadget  $f$  can transfer dominant variables from the formal parameter  $\vec{b}_i$  to its output encoding (i.e.,  $\vec{b}_i \in T_f$ ) and the encodings  $\vec{x}_1, \dots, \vec{x}_k$  in  $\vec{y} = f(\vec{x}_1, \dots, \vec{x}_k)$  are mutually independent, then  $\vec{y}$  is dominated by the dominant encodings of  $\vec{x}_i$ , namely  $\lambda(\vec{x}_i) \subseteq \lambda(\vec{y})$ .*

**PROOF.** Suppose the encoding  $\vec{x}_i$  is dominated by an encoding  $\vec{b}$  and the encodings  $\vec{x}_1, \dots, \vec{x}_k$  are mutually independent. Since the gadget  $f$  can transfer dominant variables from  $\vec{b}_i$  to its output encoding  $\vec{o}$  (i.e.,  $\vec{b}_i \in \lambda(\vec{o})$  or  $\vec{b}_i \in T_f$ ), and  $\vec{y} = f(\vec{x}_1, \dots, \vec{x}_k)$ , we get that  $\vec{y}$  is dominated by the encoding  $\vec{b}$ , because the values of the encodings  $\vec{x}_i$  and  $\vec{b}_i$  (respectively,  $\vec{o}$  and  $\vec{y}$ ) are the same.  $\square$

Remark that it is necessary to assume that the actual parameters are mutually independent in Proposition 4.10, because if shares between different actual parameters are not mutually independent, the computation of two such shares may not contain any dominant variables.

*Example 4.11.* Consider the gadget XOR in the running example. It can transfer dominant variables from the formal parameters  $\vec{a}$  and  $\vec{b}$  to the output encoding  $\vec{c}$  (i.e.,  $T_{\text{XOR}} = \{\vec{a}, \vec{b}\}$ , cf. Example 4.6). For the gadget call statement  $\vec{c} = \text{XOR}(\vec{b}, \vec{e})$  in the gadget XORMULTI, the encodings  $\vec{b}$  and  $\vec{e}$  are mutually independent, thus we have  $\lambda(\vec{b}) \subseteq \lambda(c)$  and  $\lambda(\vec{e}) \subseteq \lambda(c)$ .

In contrast, for the gadget call  $\vec{y} = \text{XOR}(\vec{x}_1, \vec{x}_2)$  such that  $\vec{x}_1 = \vec{x}_2$  (i.e.,  $\vec{x}_1$  and  $\vec{x}_2$  are not mutually independent),  $\vec{y}$  is not a dominated encoding because the shares  $\vec{y}[1]$  and  $\vec{y}[2]$  of  $\vec{y}$  are 0 even though  $T_{\text{XOR}} = \{\vec{a}, \vec{b}\}$ .  $\square$

To decide whether the actual parameters in a gadget call are mutually independent, we propose a sufficient condition based on the function  $\lambda$ . Intuitively, for the gadget call  $\vec{y} = \text{XOR}(\vec{x}_1, \vec{x}_2)$ , if  $\lambda(\vec{x}_1) = \{\vec{a}\}$ ,  $\lambda(\vec{x}_2) = \{\vec{b}\}$ , and  $\vec{a}$  and  $\vec{b}$  are mutually independent, then  $\vec{x}_1$  and  $\vec{x}_2$  are mutually independent as well. Formally, we have the following sufficient condition.

**Sufficient Condition.** Let  $\Psi(\vec{x}_1, \dots, \vec{x}_k)$  be a predicate that holds only if  $\lambda(\vec{x}_i) \neq \emptyset$  for each  $i$  and there exists a set of encodings from  $\bigcup_{i=1}^k \lambda(\vec{x}_i)$ , one encoding  $\vec{z}_i$  per set  $\lambda(\vec{x}_i)$ , such that the encodings  $\vec{z}_i$ 's are mutually independent.

**PROPOSITION 4.12.** *The encodings  $\vec{x}_1, \dots, \vec{x}_k$  are mutually independent if they are distinct and either  $\Psi(\vec{x}_1, \dots, \vec{x}_k)$  or  $\vec{x}_1 \in \lambda(\vec{x}_1), \dots, \vec{x}_k \in \lambda(\vec{x}_k)$ .*

**PROOF.** Suppose  $\vec{x}_1, \dots, \vec{x}_k$  are distinct encodings. On the one hand, if  $\vec{x}_1 \in \lambda(\vec{x}_1), \dots, \vec{x}_k \in \lambda(\vec{x}_k)$ , then  $\vec{x}_i$  must be the return of a gadget call that can generate dominant variables; meanwhile  $\vec{x}_i$  and  $\vec{x}_j$  are different returns. Thus, the distributions of  $\vec{x}_1, \dots, \vec{x}_k$  are mutually independent. On the other hand, if  $\Psi(\vec{x}_1, \dots, \vec{x}_k)$  holds, then the distributions of  $\vec{x}_i$ 's are the same as that of encodings  $\vec{z}_i$ 's. Thus, if  $\vec{z}_i$ 's are mutually independent, then  $\vec{x}_1, \dots, \vec{x}_k$  are mutually independent.  $\square$

By iteratively applying Propositions 4.8 and 4.10, a function  $\lambda$  can be computed from which we can under-approximate the summary  $T_g$  of a composite gadget  $g$ . Here, we demonstrate the general intuition on an example, as both the function  $\lambda$  and summary  $T_g$  are computed within the algorithm for inferring pre-conditions of composite gadgets (cf. Section 5.3).

*Example 4.13.* Let us consider the gadget XORMULTI in the running example. Initially,  $\lambda(\vec{a}) = \lambda(\vec{b}) = \emptyset$ :

- (1) For the gadget call  $\vec{e} = \text{Refresh}(\vec{a})$ , since  $\Psi(\vec{a})$  does not hold and  $T_{\text{Refresh}} = \{\vec{a}, \vec{e}\}$  (cf. Example 4.6), we have  $\lambda(\vec{e}) = \{\vec{e}\}$  according to Proposition 4.8 (cf. Example 4.9).
- (2) For the gadget call  $\vec{c} = \text{XOR}(\vec{b}, \vec{e})$ , since  $\Psi(\vec{b}, \vec{e})$  does not hold and  $T_{\text{XOR}} = \{\vec{a}, \vec{b}\}$  (cf. Example 4.6), we have  $\lambda(\vec{c}) = \emptyset$ .
- (3) For the gadget call  $\vec{d} = \text{UMA}(\vec{e}, \vec{c})$ , since  $\lambda(\vec{e}) = \{\vec{e}\}$  and  $\lambda(\vec{c}) = \emptyset$ ,  $\Psi(\vec{e}, \vec{c})$  does not hold. As  $T_{\text{UMA}} = \{(t_6, t_8)\}$  (cf. Example 4.6), we have  $\lambda(\vec{d}) = \{\vec{d}\}$  according to Proposition 4.8.

Finally, we obtain that  $T_{\text{XORMULTI}} = \lambda(\vec{d}) = \{\vec{d}\}$ , namely the gadget `XORMULTI` can generate dominant variables itself, and no dominant variables can be transferred from its parameters  $\vec{a}$  and  $\vec{b}$  to the output encoding. Note that in this case whether  $\vec{a}$  and  $\vec{b}$  are dominated encodings and whether they are mutually independent are unknown, thus we set  $\lambda(\vec{a}) = \lambda(\vec{b}) = \emptyset$ . If the gadget `XORMULTI` is called in another gadget where  $\lambda(\vec{a})$  and  $\lambda(\vec{b})$  are non-empty,  $\lambda$  may have different values (cf. Example 4.15).  $\square$

### 4.3 Application of Dominated Variables

Dominated variables can help reduce the size of the pre-condition  $\mathbb{I}$  when computing  $\mathbb{I}$  for a composite gadget. More specifically, if the actual parameters of a gadget call have random dominant encodings, they have uniform distributions. Thus, when instantiating  $\mathbb{I}$ , the corresponding formal parameters can be instantiated by an empty set rather than the actual parameters. Therefore, Proposition 3.4 can be refined as follows. Let  $\mathbb{I}_f = \mathbb{I}_g[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@_{\ell_g} \cup \mathbb{I}_h[\emptyset/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@_{\ell_h}$  for the composite gadget  $f$ :

$$f(\vec{x}_1, \dots, \vec{x}_m)\{\vec{y}_1 = g(\vec{x}_1, \dots, \vec{x}_m); \vec{z} = h(\vec{y}_1, \vec{x}_2, \dots, \vec{x}_n); \text{return } \vec{z};\},$$

where the gadget  $g(\vec{a}_1, \dots, \vec{a}_m)\{\dots\}$  is  $\mathbb{I}_g$ -NI and the gadget  $h(\vec{b}_1, \dots, \vec{b}_n)\{\dots\}$  is  $\mathbb{I}_h$ -NI.

**PROPOSITION 4.14.** *The gadget  $f$  is  $\mathbb{I}_f$ -NI if there exists  $\vec{x} \in \lambda(\vec{y}_1)$  such that  $\vec{x} \in \lambda(\vec{x})$ .*

**PROOF.** The proof mainly follows the lines of the proof of Proposition 3.4.

Suppose  $\vec{x} \in \lambda(\vec{y}_1)$  such that  $\vec{x} \in \lambda(\vec{x})$ . From  $\vec{x} \in \lambda(\vec{x})$ , we get that  $\vec{x}$  is the return of a gadget call that can generate dominant variables itself, namely each share  $\vec{x}[j]$  of the encoding  $\vec{x}$  is dominated by a random variable. By Proposition 4.2, each share  $\vec{x}[j]$  of the encoding  $\vec{x}$  has a uniform distribution. Since  $\vec{x} \in \lambda(\vec{y}_1)$ , we get that each share  $\vec{y}_1[j]$  of the encoding  $\vec{y}_1$  is dominated by a random variable and has a uniform distribution.

To prove that the gadget  $f$  is  $\mathbb{I}_f$ -NI, it suffices to prove that every observable variable  $x$  of  $f_{in}$  is  $I$ -simulatable for some variable set  $I \in \mathbb{I}_f$ . We will start by examining the inlined versions of the variables from the gadgets  $g$  and  $h$ , then move on to the observable variables defined in the gadget  $f$ :

- Consider an inlined version  $x@_{\ell_g}$  of a variable  $x$  from the gadget  $g$ . Since the gadget  $g$  is  $\mathbb{I}_g$ -NI, there exists a variable set  $I \in \mathbb{I}_g$  such that the variable  $x$  in the gadget  $g$  is  $I$ -simulatable. This implies that the variable  $x@_{\ell_g}$  in the gadget  $f_{in}$  is  $I[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@_{\ell_g}$ -simulatable. The result immediately follows from the fact that  $I[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@_{\ell_g} \in \mathbb{I}_g[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@_{\ell_g} \subseteq \mathbb{I}_f$ .
- Consider an inlined version  $x@_{\ell_h}$  of a variable  $x$  from the gadget  $h$ . We can get that the variable  $x@_{\ell_h}$  in the gadget  $f_{in}$  is  $I[\vec{y}_1/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@_{\ell_h}$ -simulatable for  $I \in \mathbb{I}_h$ . Moreover, if the variable set  $I$  does not involve any shares of the encoding  $\vec{b}_1$ , then  $x@_{\ell_h}$  is  $I[\emptyset/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@_{\ell_h}$ -simulatable. Otherwise, the variable set  $I$  contains some shares of the encoding  $\vec{b}_1$ , and since all shares of the encoding  $\vec{b}_1$  have the same uniform distribution,

we can get that  $x@_{\ell_h}$  is  $I[\emptyset/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@_{\ell_h}$ -simulatable as well. The result follows from the fact that  $I[\emptyset/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@_{\ell_h} \in \mathbb{I}_h[\emptyset/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@_{\ell_h} \subseteq \mathbb{I}_f$ .

- Consider an observable variable  $x$  defined in the gadget  $f$ . The variable  $x$  must be an actual parameter or a return value of one of the gadget call statements  $\vec{y}_1 = g(\vec{x}_1, \dots, \vec{x}_m)$  or  $\vec{z} = h(\vec{y}_1, \vec{x}_2, \dots, \vec{x}_n)$ .

If  $x$  is an actual parameter of  $\vec{y}_1 = g(\vec{x}_1, \dots, \vec{x}_m)$  or  $\vec{z} = h(\vec{y}_1, \vec{x}_2, \dots, \vec{x}_n)$ , let  $c$  be the corresponding formal parameter of  $x$ . Then the variable  $c$  in the gadget  $g$  is  $I$ -simulatable for some  $I \in \mathbb{I}_g$  or the variable  $c$  in the gadget  $h$  is  $I$ -simulatable for some  $I \in \mathbb{I}_h$ . Similarly, if the variable set  $I$  does not contain any shares of the encoding  $\vec{b}_1$ , we get that the variable  $x$  in the gadget  $f_{in}$  is  $I'$ -simulatable, where  $I'$  is  $I[\vec{x}_1/\vec{a}_1, \dots, \vec{x}_m/\vec{a}_m]@_{\ell_g}$  or  $I[\emptyset/\vec{b}_1, \vec{x}_2/\vec{b}_2, \dots, \vec{x}_n/\vec{b}_n]@_{\ell_h}$ , since  $x$  and  $c$  always have the same value. Otherwise, the shares of the encoding  $\vec{b}_1$  contained in the variable set  $I$  must have the same uniform distribution, and we get that  $x$  in the gadget  $f_{in}$  is  $I'$ -simulatable as well.

If  $x$  is a return value of  $\vec{y}_1 = g(\vec{x}_1, \dots, \vec{x}_m)$  or  $\vec{z} = h(\vec{y}_1, \vec{x}_2, \dots, \vec{x}_n)$ , let  $c$  be the corresponding return variable of  $x$  in the gadget  $g$  or  $h$ . We have proved that the variable  $c@_{\ell_g}$  (respectively,  $c@_{\ell_h}$ ) in the gadget  $f_{in}$  is  $I$ -simulatable if  $c$  is a return variable of  $g$  (respectively,  $h$ ) for some  $I \in \mathbb{I}_f$ . Since  $x$  and  $c$  always have the same value, we get that the variable  $x$  in the gadget  $f_{in}$  is  $I$ -simulatable.

This completes the proof.  $\square$

*Example 4.15.* Let us consider the gadget call to XORMULTI in the running example. Since  $\vec{a}$  and  $\vec{b}$  are return encodings of two calls to the encoding gadget (i.e., Encoding), we have  $\lambda(\vec{a}) = \{\vec{a}\}$ ,  $\lambda(\vec{b}) = \{\vec{b}\}$ , and  $\vec{a}$  and  $\vec{b}$  are mutually independent. We show how to compute  $\mathbb{I}_{\text{XORMULTI}}$  in this context:

- (1) For  $\vec{e} = \text{Refresh}(\vec{a})$ , as  $\lambda(\vec{a}) = \{\vec{a}\}$  and  $T_{\text{Refresh}} = \{\vec{a}, \vec{c}\}$ , we get that  $\Psi(\vec{a})$  holds,  $\lambda(\vec{e}) = \{\vec{a}, \vec{e}\}$ , and  $\mathbb{I}_{\text{Refresh}}@8@4 = \mathbb{I}_{\text{Refresh}}[\emptyset/\vec{a}] = \emptyset$  according to Propositions 4.8, 4.10, and 4.14.
- (2) For  $\vec{c} = \text{XOR}(\vec{b}, \vec{e})$ , as  $\lambda(\vec{b}) = \{\vec{b}\}$ ,  $\lambda(\vec{e}) = \{\vec{a}, \vec{e}\}$ , and  $T_{\text{XOR}} = \{\vec{a}, \vec{b}\}$ , we get  $\Psi(\vec{b}, \vec{e})$  holds,  $\lambda(\vec{c}) = \{\vec{a}, \vec{b}, \vec{e}\}$ , and  $\mathbb{I}_{\text{XOR}}@9@4 = \mathbb{I}_{\text{XOR}}[\emptyset/\vec{a}, \emptyset/\vec{b}] = \emptyset$ .
- (3) For  $\vec{d} = \text{UMA}(\vec{e}, \vec{c})$ , as  $\lambda(\vec{e}) = \{\vec{a}, \vec{e}\}$  and  $\lambda(\vec{c}) = \{\vec{a}, \vec{b}, \vec{e}\}$ , we get  $\Psi(\vec{e}, \vec{c})$  holds,  $\lambda(\vec{d}) = \vec{d}$ , and  $\mathbb{I}_{\text{UMA}}@10@4 = \mathbb{I}_{\text{UMA}}[\emptyset/\vec{a}, \emptyset/\vec{b}] = \emptyset$ .

Finally, we obtain that  $\mathbb{I}_{\text{XORMULTI}} = \emptyset$ . Thus, XORMULTI is first-order probing secure. Compared with the result in Example 4.13 where  $\lambda(\vec{a}) = \lambda(\vec{b}) = \emptyset$ , we only deduced that  $\lambda(\vec{e}) = \{\vec{e}\}$  and  $\lambda(\vec{c}) = \emptyset$ . Compared with the result in Example 3.5 where  $\mathbb{I}_{\text{XORMULTI}} = \{\{\vec{a}[i]\}, \{\vec{b}[i], \vec{e}[i]\}, \{\vec{e}[i], \vec{c}[j]\} \mid 1 \leq i, j \leq 2\}$ ,  $\mathbb{I}_{\text{XORMULTI}}$  turns to  $\emptyset$  using the dominant variables of the actual parameters  $\vec{a}$  and  $\vec{b}$ .  $\square$

## 5 ALGORITHMIC VERIFICATION

In this section, we first present a sound proof system for checking simulatability. Then we introduce algorithms to infer pre-conditions for simple gadgets and composite gadgets by utilizing the proof system and composition rules.

### 5.1 A Sound Proof System for Checking Simulatability

We first show how to use random dominant variables to simplify computations, which is leveraged to derive valid judgments in our proof system. Given a sub-expression  $e \in \text{Sub}(x)$  and a random variable  $r \in \text{Var}(\mathcal{E}(x))$ , let  $x[r/e]$  denote the new variable  $x'$  such that  $\mathcal{E}(x')$  is obtained by

$$\boxed{\frac{x \in I \vee \text{Var}(\mathcal{E}(x)) \subseteq I \cup X_r^{fin}}{\vdash I \rightsquigarrow x} \text{ (SUPP)} \qquad \frac{x \xrightarrow{(e,r)} x' \quad \vdash I \rightsquigarrow x'}{\vdash I \rightsquigarrow x} \text{ (DOM)}}$$

Fig. 6. Proof rules.

replacing  $e$  by  $r$  in  $\mathcal{E}(x)$ . (Note that  $\text{Sub}(x)$  denotes the set of the sub-expressions of  $\mathcal{E}(x)$ .) For each sub-expression  $e \in \text{Sub}(x)$ , if  $r \in \text{DomR}(e)$  and  $r$  does not occur anywhere else in  $\mathcal{E}(x)$ ,  $x$  can be simplified to  $x' = x[r/e]$ , denoted as  $x \xrightarrow{(e,r)} x'$ . This process can be repeated until there does not exist such random dominant variable  $r$  and sub-expression  $e$ . By Proposition 4.2,  $x$  is  $I$ -simulatable iff  $x'$  is  $I$ -simulatable.

Given a gadget  $f$ , the judgment is in the form of

$$\vdash I \rightsquigarrow x,$$

where  $I \subseteq X_a^f \cup X^{fin}$  and  $x \in X^{fin}$ . The judgment  $\vdash I \rightsquigarrow x$  is valid iff the variable  $x$  in the gadget  $f$  is  $I$ -simulatable.

Figure 6 shows two proof rules for deriving valid judgment  $\vdash I \rightsquigarrow x$ . The first rule (i.e., SUPP) exploits syntactic information and states that if the variable  $x$  is in  $I$  or the support variables of  $\mathcal{E}(x)$  are either random variables or in  $I$ , then  $\vdash I \rightsquigarrow x$  is valid. Another rule (i.e., DOM) makes use of semantic information. If a variable  $x$  can be simplified to  $x'$  using the random dominant variable  $r$  and  $\vdash I \rightsquigarrow x'$  is valid, then  $\vdash I \rightsquigarrow x$  is valid. This proof system will be used to prove  $\mathbb{I}$ -NI of simple gadgets in the next sub-section.

**THEOREM 5.1.** *If  $\vdash I \rightsquigarrow x$  can be derived by the proof system, then the variable  $x$  in the gadget  $f$  is  $I$ -simulatable.*

**PROOF.** It suffices to prove the soundness of the preceding two rules:

- *Rule (SUPP):* Suppose the premise  $x \in I \vee \text{Var}(\mathcal{E}(x)) \subseteq I \cup X_r^{fin}$  holds. If  $x \in I$ , since  $x$  is  $x$ -simulatable, by Proposition 2.3, we get that  $x$  is  $I$ -simulatable. If  $\text{Var}(\mathcal{E}(x)) \subseteq I \cup X_r^{fin}$ , then the values of the variables in  $\text{Var}(\mathcal{E}(x)) \setminus X_r^{fin}$  can be directly obtained from the values of the variables in  $I$ . Since the values of the variables in  $\text{Var}(\mathcal{E}(x)) \cap X_r^{fin}$  are uniformly sampled, knowing the values of  $I$  suffices to simulate the distribution of  $x$ , hence  $x$  is  $I$ -simulatable.
- *Rule (DOM):* Suppose  $x \xrightarrow{(e,r)} x'$ . By Proposition 4.2, we have that  $x$  is  $I$ -simulatable iff  $x'$  is  $I$ -simulatable. The result immediately follows.

We conclude the proof.  $\square$

**Example 5.2.** Consider the gadget UMA in the running example. From  $\mathcal{E}(t_1) = \vec{a}[1] \odot \vec{b}[1]$ ,  $\mathcal{E}(t_2) = \vec{a}[2] \odot \vec{b}[2]$ ,  $\mathcal{E}(t_3) = \vec{a}[1] \odot \vec{b}[2]$ , and  $\mathcal{E}(t_4) = \vec{a}[2] \odot \vec{b}[1]$ , by applying Rule (SUPP), we can deduce that  $\vdash \{\vec{a}[1], \vec{b}[1]\} \rightsquigarrow t_1$ ,  $\vdash \{\vec{a}[2], \vec{b}[2]\} \rightsquigarrow t_2$ ,  $\vdash \{\vec{a}[1], \vec{b}[2]\} \rightsquigarrow t_3$ , and  $\vdash \{\vec{a}[2], \vec{b}[1]\} \rightsquigarrow t_4$  are valid, thus  $t_1$  is  $\{\vec{a}[1], \vec{b}[1]\}$ -simulatable,  $t_2$  is  $\{\vec{a}[2], \vec{b}[2]\}$ -simulatable,  $t_3$  is  $\{\vec{a}[1], \vec{b}[2]\}$ -simulatable, and  $t_4$  is  $\{\vec{a}[2], \vec{b}[1]\}$ -simulatable.

Consider a variable  $x$  such that  $\mathcal{E}(x) = (a \oplus r_1 \oplus r_2) \odot (b \oplus r_1)$ , where  $r_1$  and  $r_2$  are random variables. Without applying Rule (DOM), we can only deduce that  $x$  is  $\{a, b\}$ -simulatable. By applying Rule (DOM), the sub-expression  $(a \oplus r_1 \oplus r_2)$  can be replaced by the random variable  $r_2$ , leading to the simplified expression  $\mathcal{E}(x') = r_2 \odot (b \oplus r_1)$ , which further can be simplified to  $\mathcal{E}(x'') = r_2 \odot r_1$ . Obviously,  $\vdash \emptyset \rightsquigarrow x''$  is valid, thus we deduce that all of  $x$ ,  $x'$ , and  $x''$  are  $\emptyset$ -simulatable.  $\square$



**ALGORITHM 1:** Checking simple gadget

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```

1: procedure SGADGET( $f$ )
2:   if storedInfo( $f$ ) exists then
3:     return storedInfo( $f$ )
4:   Let  $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m$  be the formal parameters of  $f$ 
5:   Let  $\vec{o}$  be the output encoding of  $f$ 
6:    $\mathbb{I} = \{I \mid I = \text{Var}(\mathcal{E}(x)) \subseteq X_a^f \wedge x \in (X_a^f \cup X^f)\}$ 
7:    $T = \emptyset$ 
8:    $\mathbb{I} = \mathbb{I} \setminus \{I \in \mathbb{I} \mid \exists I' \in \mathbb{I}. I \subset I'\}$ 
9:   for each variable  $x \in X^f$  do
10:    if  $\nexists I \in \mathbb{I}$  s.t.  $\vdash I \rightsquigarrow x$  then
11:       $\mathbb{I} = \mathbb{I} \cup \{\{x\}\}$ 
12:   for each  $\vec{a}_i \in \{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m\}$  do
13:     if  $f$  can transfer dominant variables from  $\vec{a}_i$  to  $\vec{o}$  then
14:        $T = T \cup \{\vec{a}_i\}$ 
15:   if  $f$  can generate dominant variables then
16:      $T = T \cup \{\vec{o}\}$ 
17:   storedInfo( $f$ ) =  $(\mathbb{I}, T)$ 
18:   return  $(\mathbb{I}, T)$ 

```

---

**5.2 Inferring Pre-Conditions of Simple Gadgets**

Algorithm 1 shows how to infer pre-conditions  $\mathbb{I}$  for simple gadgets by leveraging the sound proof system to check simulatability. SGADGET takes a simple gadget  $f$  as input, and outputs  $(\mathbb{I}, T)$  such that  $f$  is  $\mathbb{I}$ -NI and  $T$  is the summary of the gadget  $f$ . We use the mapping storedInfo to store the checking results. Thus, if storedInfo( $f$ ) exists, it immediately returns the result stored in storedInfo. Otherwise, it computes  $(\mathbb{I}, T)$  for the gadget  $f$ .

The pre-condition  $\mathbb{I}$  is initialized (line 6) by the following set:

$$\{I \mid I = \text{Var}(\mathcal{E}(x)) \subseteq X_a^f \wedge x \in X_a^f \cup X^f\}.$$

Intuitively, the computation  $\mathcal{E}(x)$  of a variable  $x$  whose support variables are all the shares of input encodings cannot be simplified. Thus,  $I = \text{Var}(\mathcal{E}(x))$  should be added into the pre-condition  $\mathbb{I}$ . After initializing  $\mathbb{I}$ , to keep  $\mathbb{I}$  as small as possible, if there exists a set  $I \in \mathbb{I}$  such that  $I$  is the subset of  $I' \in \mathbb{I}$ , then  $I$  is removed from  $\mathbb{I}$  (line 8). For each variable  $x \in X^f$ , if there does not exist  $I \in \mathbb{I}$  such that  $\vdash I \rightsquigarrow x$  is valid via our proof rules, the set  $\{x\}$  is directly added into the pre-condition  $\mathbb{I}$  (lines 9–11). Next, it checks whether  $f$  can transfer dominant variables from the input encoding  $\vec{a}_i$  to the output encoding  $\vec{o}$  for each  $\vec{a}_i \in \{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m\}$ . By Definition 4.4, if  $f$  can transfer dominant variables from  $\vec{a}_i$  to the output encoding  $\vec{o}$ , the input encoding  $\vec{a}_i$  is added into the summary  $T$  (lines 12–14). Whether the input encoding  $\vec{a}_i$  can transfer dominant variables to the output encoding  $\vec{o}$  is checked according to Definition 4.1, aimed with the simplification  $x \xrightarrow{(e,r)} x'$ . By Definition 4.5, if  $f$  can generate dominant variables itself, the output encoding  $\vec{o}$  is added into the summary  $T$  (lines 15 and 16). Whether  $f$  can generate dominant variables itself is checked easily by checking if  $\vdash \emptyset \rightsquigarrow \vec{o}[j']$  is valid for each share  $\vec{o}[j']$  of the output encoding  $\vec{o}$ . Finally, the result  $(\mathbb{I}, T)$  is stored in storedInfo and then returned. It is easy to see that  $f$  is  $\mathbb{I}$ -NI and  $T$  is the summary of  $f$ .

*Example 5.3.* Consider the simple gadget Refresh in the running example. Algorithm 1 first initializes the set  $\mathbb{I}$  as  $\{\{\vec{a}[1]\}, \{\vec{a}[2]\}\}$  at line 6. The set  $\mathbb{I}$  cannot be simplified at line 8. At lines 9 through 11, the proof system will prove that  $\vdash \emptyset \rightsquigarrow \vec{c}[1]$  and  $\vdash \emptyset \rightsquigarrow \vec{c}[2]$  are valid, thus neither

$\vec{c}[1]$  nor  $\vec{c}[2]$  is added into the set  $\mathbb{I}$ . At lines 12 through 14, the input encoding  $\vec{a}$  is added into the summary  $T$ , because the share  $\vec{c}[i]$  of the encoding  $\vec{c}$  is dominated by only the share  $\vec{a}[i]$  of the input encoding  $\vec{a}$  for  $i = 1, 2$ . At lines 15 and 16, the output encoding  $\vec{c}$  is added into the summary  $T$ , because  $\vdash \emptyset \rightsquigarrow \vec{c}[1]$  and  $\vdash \emptyset \rightsquigarrow \vec{c}[2]$  are valid. Finally, Algorithm 1 returns the pair  $(\{\{\vec{a}[1]\}, \{\vec{a}[2]\}\}, \{\vec{a}, \vec{c}\})$ .

Consider the gadget XOR in the running example. Algorithm 1 first initializes the set  $\mathbb{I}$  as  $\{\{\vec{a}[1]\}, \{\vec{a}[2]\}, \{\vec{b}[1]\}, \{\vec{b}[2]\}, \{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}\}$  at line 6. At line 8, the set  $\mathbb{I}$  is simplified to  $\{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}\}$ . At lines 9 through 11, the proof system will prove that  $\vdash \{\vec{a}[1], \vec{b}[1]\} \rightsquigarrow \vec{c}[1]$  and  $\vdash \{\vec{a}[2], \vec{b}[2]\} \rightsquigarrow \vec{c}[2]$  are valid, thus neither  $\vec{c}[1]$  nor  $\vec{c}[2]$  is added into the set  $\mathbb{I}$ . At lines 12 through 14, the input encodings  $\vec{a}$  and  $\vec{b}$  are added into the summary  $T$ , because the share  $\vec{c}[i]$  of the encoding  $\vec{c}$  is dominated by only the share  $\vec{a}[i]$  of  $\vec{a}$  and the share  $\vec{b}[i]$  of  $\vec{b}$  for  $i = 1, 2$ . At lines 15 and 16, the output encoding  $\vec{c}$  cannot be added into the summary  $T$ , because neither  $\vdash \emptyset \rightsquigarrow \vec{c}[1]$  nor  $\vdash \emptyset \rightsquigarrow \vec{c}[2]$  is valid. Finally, Algorithm 1 returns the pair  $(\{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}\}, \{\vec{a}, \vec{b}\})$ .

Similarly, Algorithm 1 returns  $(\{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}, \{\vec{a}[1], \vec{b}[2]\}, \{\vec{a}[2], \vec{b}[1]\}\}, \{(t_6, t_8)\})$  for the gadget UMA.

These results are consistent with the ones given in Example 3.3, namely Refresh is  $\mathbb{I}_{\text{Refresh-NI}}$ , XOR is  $\mathbb{I}_{\text{XOR-NI}}$ , and UMA is  $\mathbb{I}_{\text{UMA-NI}}$ , where

$$\begin{aligned} - \mathbb{I}_{\text{Refresh}} &= \{\{\vec{a}[1]\}, \{\vec{a}[2]\}\}; \\ - \mathbb{I}_{\text{XOR}} &= \{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}\}; \\ - \mathbb{I}_{\text{UMA}} &= \{\{\vec{a}[1], \vec{b}[1]\}, \{\vec{a}[2], \vec{b}[2]\}, \{\vec{a}[1], \vec{b}[2]\}, \{\vec{a}[2], \vec{b}[1]\}\}. \end{aligned} \quad \square$$

### 5.3 Inferring Pre-Conditions for Composite Gadgets

We propose the two procedures GADGET and CGADGET in Algorithm 2, where the procedure GADGET takes a gadget  $f$  and a function  $\lambda$  as input and outputs the result  $(\mathbb{I}, T)$  such that  $f$  is  $\mathbb{I}$ -NI and  $T$  is the summary of the gadget  $f$ . If  $f$  is a simple gadget, then the procedure GADGET calls the procedure SGADGET at line 2 (cf. Algorithm 1) and returns the result immediately. Otherwise, it calls the procedure CGADGET at line 3 that takes a composite gadget  $f$  and  $\lambda$  as input and outputs  $(\mathbb{I}, T)$  such that the gadget  $f$  is  $\mathbb{I}$ -NI and  $T$  is the summary of the gadget  $f$ .

Recall that  $\lambda$  is a function used for computing summaries of gadgets (cf. Section 4.2.2). For each encoding  $\vec{a}$ ,  $\lambda(\vec{a})$  contains all the dominant encodings of  $\vec{a}$ , and moreover  $\vec{a} \in \lambda(\vec{a})$  only if  $\vec{a}$  is the return of a gadget call that can generate dominant variables. However,  $\lambda$  is unknown in advance, thus we will compute the function  $\lambda$  during analysis for each composite gadget. We also use the mapping storedInfo to store the checking result, namely storedInfo( $f, \lambda$ ) records the result of CGADGET( $f, \lambda$ ), where  $\lambda$  initially contains only the dominant encodings of the formal parameters of the gadget  $f$  and thus can be seen as the calling context of the gadget  $f$ . We note that the same gadget may be called multiple times with different calling contexts, thus  $\lambda$  is involved in storedInfo.

The procedure CGADGET first checks if storedInfo( $f, \lambda$ ) exists or not, and it returns storedInfo( $f, \lambda$ ) if it exists (line 7). Otherwise, it reasons about the gadget  $f$  with the function  $\lambda$ . First, the pre-condition  $\mathbb{I}$  is set to  $\emptyset$  (line 8) and  $\lambda_1$  is set to  $\lambda$  as a backup of calling context  $\lambda$  (line 9). For each gadget call  $\vec{y} = g(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_m)$  at call-site  $\ell_g$ , it computes the calling context  $\lambda'$  based on  $\lambda(\vec{x}_i)$  (lines 15–17), namely for each formal parameter  $\vec{b}_i$ ,  $\lambda'(\vec{b}_i)$  is  $\{\vec{b}_i\}$  if  $\exists \vec{x} \in \lambda(\vec{x}_i) \cap \lambda(\vec{x})$ , and otherwise  $\emptyset$ , where  $\vec{x} \in \lambda(\vec{x}_i)$  indicates that the actual parameter  $\vec{x}_i$  is dominated by the encoding  $\vec{x}$  and  $\vec{x} \in \lambda(\vec{x})$  indicates that  $\vec{x}$  is the return of a gadget call to a gadget that generates dominant variables.

After constructing  $\lambda'$ , the result  $(\mathbb{I}', T')$  of the gadget  $g$  under  $\lambda'$  is obtained by invoking GADGET( $g, \lambda'$ ) (line 18). Then, by Proposition 4.10, if  $\vec{x}_1, \dots, \vec{x}_m$  are mutually independent and

**ALGORITHM 2:** Checking composite gadget

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```

1: procedure GADGET( $f, \lambda$ )
2:   if  $f$  is a simple gadget then  $(\mathbb{I}, T) = \text{SGADGET}(f)$ 
3:   else  $(\mathbb{I}, T) = \text{CGADGET}(f, \lambda)$ 
4:   return  $(\mathbb{I}, T)$ 
5: procedure CGADGET( $f, \lambda$ )
6:   if storedInfo( $f, \lambda$ ) exists then
7:     return storedInfo( $f, \lambda$ )
8:    $\mathbb{I} = \emptyset$ 
9:    $\lambda_1 = \lambda$ 
10:  Let  $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n$  be the formal parameters of  $f$ 
11:  Let  $\vec{o}_1$  be the output encoding of  $f$ 
12:  for each gadget call  $\vec{y} = g(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_m)$  at  $\ell_g$  from the first to the last do
13:    Let  $\vec{b}_1, \dots, \vec{b}_m$  be the formal parameters of  $g$ 
14:    Let  $\vec{o}_2$  be the output encoding of  $g$ 
15:    for  $i \in \{1, \dots, m\}$  do
16:      if  $\exists \vec{x} \in \lambda(\vec{x}_i) \cap \lambda(\vec{x})$  then  $\lambda'(\vec{b}_i) = \{\vec{b}_i\}$ 
17:      else  $\lambda'(\vec{b}_i) = \emptyset$ 
18:       $(\mathbb{I}', T') = \text{GADGET}(g, \lambda')$ 
19:      if  $\vec{x}_1, \dots, \vec{x}_m$  are distinct then
20:        if  $\Psi(\vec{x}_1, \dots, \vec{x}_m)$  or  $\vec{x}_1 \in \lambda(\vec{x}_1), \dots, \vec{x}_m \in \lambda(\vec{x}_m)$  then
21:          for  $i \in \{1, \dots, m\}$  do
22:            if  $\vec{b}_i \in T'$  then  $\lambda(\vec{y}) = \lambda(\vec{y}) \cup \lambda(\vec{x}_i)$ 
23:          if  $\vec{o}_2 \in T'$  then
24:             $\lambda(\vec{y}) = \lambda(\vec{y}) \cup \{\vec{y}\}$ 
25:          for  $i \in \{1, \dots, m\}$  do
26:             $\vec{x}'_i = (\exists \vec{x} \in \lambda(\vec{x}_i) \cap \lambda(\vec{x}) ? \emptyset : \vec{x}_i)$ 
27:           $\mathbb{I} = \mathbb{I} \cup \mathbb{I}'[\vec{x}'_1/\vec{b}_1, \dots, \vec{x}'_m/\vec{b}_m]@_{\ell_g}$ 
28:           $T = \lambda(\vec{o}_1)$ 
29:          if  $T \setminus \{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n, \vec{o}_1\} \neq \emptyset$  then
30:             $T = (T \cap \{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n, \vec{o}_1\}) \cup \{\vec{o}_1\}$ 
31:          storedInfo( $f, \lambda_1$ ) =  $(\mathbb{I}, T)$ 
32:          return  $(\mathbb{I}, T)$ 

```

---

(either  $\Psi(\vec{x}_1, \dots, \vec{x}_m)$  holds or  $\vec{x}_1 \in \lambda(\vec{x}_1), \dots, \vec{x}_m \in \lambda(\vec{x}_m)$ ), for each formal parameter  $\vec{b}_i \in T'$ , the set  $\lambda(\vec{x}_i)$  is merged into  $\lambda(\vec{y})$  (line 22). Next, by Proposition 4.8, if  $g$  can generate dominant variables itself (i.e.,  $\vec{o}_2 \in T'$ ),  $\vec{y}$  is added into  $\lambda(\vec{y})$  too (line 24). After that, the pre-condition  $\mathbb{I}$  is computed following Proposition 4.14 (line 27).

After computing  $\mathbb{I}$ , it continues to compute the summary  $T$  of the gadget  $f$  that is initialized as  $\lambda(\vec{o}_1)$ . To ensure that  $T \subseteq X_{en}^f \cup \{\vec{o}_1\}$ , it removes all internal encodings of  $f$  from the summary  $T$  and uses  $\vec{o}_1$  to indicate that  $f$  can generate dominant variables itself (cf. Proposition 4.7). Finally, the result  $(\mathbb{I}, T)$  is stored in storedInfo and returned. Note that after computing  $\mathbb{I}$ ,  $\lambda$  has been updated. Thus, we use its backup  $\lambda_1$  to store the result (line 31).

**THEOREM 5.4.** *Given a program  $P$  with the main gadget*

$$\text{main}(a_1, \dots, a_m)\{\text{enstmt}^+ \text{gstmt}^+ \text{return } \vec{b};\},$$

let  $g$  be the gadget  $g(\vec{a}_1, \dots, \vec{a}_m)\{\text{gstmt}^+ \text{ return } \vec{b};\}$  with the same gadget calls  $\text{gstmt}^+$  as  $\text{main}$ , where  $\vec{a}_1, \dots, \vec{a}_m$  are the encodings of  $a_1, \dots, a_m$  via calling an encoding gadget (e.g., `Encoding`). Let  $\lambda$  be a function such that  $\lambda(\vec{a}_i) = \{\vec{a}_i\}$  for  $\vec{a}_i \in \{\vec{a}_1, \dots, \vec{a}_m\}$ . If  $(\mathbb{I}, T) = \text{GADGET}(g, \lambda)$  and each variable set  $I \in \mathbb{I}$  is  $X_p$ -simulatable, then  $P$  is first-order probing secure.

**PROOF.** Since  $(\mathbb{I}, T) = \text{GADGET}(g, \lambda)$ , we get that  $g$  is  $\mathbb{I}$ -NI. The result immediately follows from Proposition 3.2 and the fact that  $\vec{a}_1, \dots, \vec{a}_m$  are the encodings of  $a_1, \dots, a_m$  via calling an encoding gadget (e.g., `Encoding`).  $\square$

*Example 5.5.* Consider the composite gadget `XORMULTI` in the running example that is called in the main gadget. Procedure `CGADGET` in Algorithm 2 is invoked for the gadget `XORMULTI` with  $\lambda(\vec{a}) = \{\vec{a}\}$  and  $\lambda(\vec{b}) = \{\vec{b}\}$ . During the for-loop at lines 12 through 27, the gadget call  $\vec{e} = \text{Refresh}(\vec{a})$  is first processed as follows:

- Since  $\lambda(\vec{a}) = \{\vec{a}\}$ , we have  $\lambda'(\vec{a}) = \{\vec{a}\}$  at lines 15 through 17.
- The pair  $(\{\{\vec{a}[1]\}, \{\vec{a}[2]\}\}, \{\vec{a}, \vec{c}\})$  is returned at line 18 (cf. Example 5.3).
- Since  $\lambda(\vec{a}) = \{\vec{a}\}$  and  $\vec{a} \in T' = \{\vec{a}, \vec{c}\}$ ,  $\lambda(\vec{e})$  is set to  $\lambda(\vec{e}) \cup \lambda(\vec{a})$ , that is  $\{\vec{a}\}$ , at line 22.
- Since  $\vec{c} \in T' = \{\vec{a}, \vec{c}\}$ ,  $\{\vec{e}\}$  is added into  $\lambda(\vec{e})$ , leading to  $\lambda(\vec{e}) = \{\vec{a}, \vec{e}\}$  at line 24.
- Since  $\lambda(\vec{a}) = \{\vec{a}\}$ ,  $\vec{x}'_i$  is set to  $\emptyset$  at line 26.
- Finally,  $\mathbb{I}$  is set to  $\mathbb{I} \cup \{\{\vec{a}[1]\}, \{\vec{a}[2]\}\}[\emptyset/\vec{a}]@_{\ell_g}$  that is  $\emptyset$  at line 27.

Later, the gadget calls  $\vec{e} = \text{Refresh}(\vec{a})$ ,  $\vec{c} = \text{XOR}(\vec{b}, \vec{e})$ , and  $\vec{d} = \text{UMA}(\vec{e}, \vec{c})$  are iteratively processed similar to the gadget call  $\vec{e} = \text{Refresh}(\vec{a})$ . At the exit of the for-loop at lines 12 through 27,  $\mathbb{I}$  is still  $\emptyset$  and  $\lambda(\vec{d}) = \{\vec{d}\}$ . Finally, procedure `CGADGET` returns the pair  $(\emptyset, \{d\})$ .

This result is consistent with the one given in Example 4.15, namely `XORMULTI` is  $\emptyset$ -NI.  $\square$

We remark that pre-conditions of both simple and composite gadgets can always be successfully and automatically computed without inlining gadget calls and user interactions. However, technically speaking, in the worst case, the pre-condition of a simple gadget  $g$  may contain all the input parameters and internal variables. For instance, if the computation  $\mathcal{E}(x)$  of each internal variable  $x$  uses some random variables but is not dominated by any random variables, then  $\mathbb{I} = \{\{a\} \mid a \in X_a^g\}$  at line 6 of Algorithm 1 and further the sets  $\{x\}$  for all  $x \in X^g$  will be added into the set  $\mathbb{I}$  at lines 9 through 11 of Algorithm 1. Hence, the set  $\mathbb{I}'$  obtained at line 18 of Algorithm 2 will be  $\{\{x\} \mid x \in X_a^g \cup X^g\}$ . If the worst case occurs for all of the simple gadgets, the pre-condition  $\mathbb{I}$  of a composite gadget  $f$  will contain all input parameters and internal variables of its inlined version  $f_{in}$ , which is the same as that the pre-condition of  $f$  is computed on the inlined version  $f_{in}$ . We should emphasize that the worst case never occurs in our experiments, because the computations  $\mathcal{E}(x)$  of internal variables typically either depend only on input parameters (in this case,  $\text{Var}(\mathcal{E}(x)) \subseteq X_a^f$  is added into the set  $\mathbb{I}$  at line 6 of Algorithm 1) or are perfectly masked by XORing random variables (in this case,  $\vdash \emptyset \rightsquigarrow x$  is valid at lines 9–11 of Algorithm 1 and  $\{x\}$  will not be added into  $\mathbb{I}$ ).

## 6 EXPERIMENTS

We have implemented our approach in a tool named `MASKCV`. Given a masked program and its security type annotation of input parameters, `MASKCV` first preprocesses the program by unfolding bounded for-loops and transforming into an intermediate representation in SSA form, then infers pre-conditions and verifies first-order probing security, all of which are done automatically without any user interactions. In this section, we thoroughly evaluate `MASKCV` focusing on the following research questions:

Table 2. Basic Information of Benchmarks

Name	#Gadgets	#Gadget Calls	#Annotations for QMVERIF
Sbox1	10	14	18
Sbox2	10	14	18
Sbox3	11	12	22
Sbox4	11	12	22
Sbox5	10	14	30
Sbox6	10	14	30
Sbox7	10	14	30
Sbox8	10	14	30
Sbox9	11	12	34
Sbox10	11	12	34
AES1	20	3,433	178
AES2	20	3,433	146
AES3	21	3,033	152
AES4	21	3,033	150
AES5	20	3,433	206
AES6	20	3,433	206
AES7	20	3,433	206
AES8	20	3,433	206
AES9	21	3,033	228
AES10	21	3,033	228
bitslicedSbox1	5	125	78
bitslicedSbox2	5	125	78

*RQ1*: How efficient is our approach compared to the state-of-the-art approaches?

*RQ2*: How effective are the dominated variables?

We mainly use the publicly available benchmarks from QMVERIF [53]: 10 versions of masked implementations of arithmetic AES Sbox and 10 corresponding masked implementations of full AES. We also implement two versions of bitsliced AES Sbox. The bitsliced AES Sbox was originally proposed in the work of Goudarzi and Rivain [57], which contains multiple gadget calls to the Refresh gadget. Later Belaïd et al. [18] proved that the bitsliced AES Sbox is still secure if all calls to the Refresh gadget are removed, leading to an efficient and tight bitsliced AES Sbox in terms of gadget calls to the Refresh gadget. Moreover, we notice that the efficiency in terms of the randomness of the tight bitsliced AES Sbox can be improved further by replacing the calls to the SecMult gadget by more efficient secure gadgets (i.e., Para [11] and Comp [14]). Thus, we construct two implementations of bitsliced AES Sbox bitslicedSbox1 and bitslicedSbox2 by replacing the calls to SecMult by Comp and Para, respectively. We remark that the first-order security of all of those benchmarks cannot be verified by any of the existing compositional verifiers, although some of them support the verification of higher-order security, except for QMVERIF. Details of the benchmarks are shown in Table 2, including the number of gadgets, the number of gadget calls, and the size of annotations used for QMVERIF.

All experiments were conducted on a server with Ubuntu 16.04 and an Intel Xeon CPU E5-2690 v4@2.60 GHz and 256 GB of RAM. The reported verification time includes all computation times for preprocessing, determining dominated variables, computing pre-conditions, and verifying first-order probing security based on pre-conditions.

**RQ1.** We compare with the state-of-the-art compositional verifier QMVERIF and the state-of-the-art non-compositional ones LeakageVerif [77] and SILVER [69], where LeakageVerif uses symbolic analysis and SILVER is based on BDD analysis. Note that SILVER only supports Boolean programs, thus we evaluate it only on Boolean programs (i.e., bitslicedSbox1 and bitslicedSbox2) and mark N/A on arithmetic programs. All of the tools QMVERIF, LeakageVerif, SILVER, and

Table 3. Results of Comparison with Existing

Name	Result	QMVERIF [53]				LeakageVerif [77]		SILVER [69]	MASKCV	
		Without Pre-Conditions		With Pre-Conditions		Time(s)	#checked	Time(s)	Time(s)	#checked
		Time(s)	#checked	Time(s)	#checked					
Sbox1	✓	0.01	46	0.01	19	0.04	51	N/A	0.01	22
Sbox2	✓	0.01	50	0.01	19	0.07	43	N/A	0.01	23
Sbox3	✓	0.01	70	0.01	60	0.08	65	N/A	0.01	68
Sbox4	✓	0.01	68	0.01	60	0.09	69	N/A	0.01	67
Sbox5	✓	0.01	110	0.01	42	0.15	115	N/A	0.01	45
Sbox6	✓	0.01	122	0.01	42	0.10	103	N/A	0.01	48
Sbox7	✓	0.01	118	0.01	50	0.17	167	N/A	0.01	48
Sbox8	✓	0.01	130	0.01	50	0.19	173	N/A	0.01	51
Sbox9	✓	0.01	178	0.01	150	0.16	162	N/A	0.01	174
Sbox10	✓	0.01	172	0.01	150	0.16	162	N/A	0.01	171
AES1	✓	315	11,142	0.10	2,182	559	10,706	N/A	0.02	29
AES2	✓	197	11,942	0.10	2,183	561	10,706	N/A	0.02	30
AES3	✓	559	15,942	0.10	2,393	1,836	14,706	N/A	0.02	75
AES4	✓	560	15,542	0.10	2,392	2,144	14,706	N/A	0.02	74
AES5	✓	2,670	24,724	0.40	4,214	T.O.	N/A	N/A	0.03	55
AES6	✓	3,505	27,124	0.40	4,214	T.O.	N/A	N/A	0.03	58
AES7	✓	2,933	26,324	0.50	4,430	T.O.	N/A	N/A	0.03	58
AES8	✓	3,130	28,724	0.50	4,430	T.O.	N/A	N/A	0.03	61
AES9	✓	2,929	38,324	0.20	3,786	O.O.M.	N/A	N/A	0.03	184
AES10	✓	3,064	37,124	0.20	3,786	O.O.M.	N/A	N/A	0.04	181
bitslicedSbox1	✓	0.02	961	0.02	451	9.30	873	O.O.M.	0.01	27
bitslicedSbox2	✓	0.02	1,057	0.02	453	8.47	937	T.O.	0.01	30

Approaches Time(s) denotes the overall verification time in seconds (s), T.O. denotes time-out (6 hours), O.O.M. denotes out of memory, and #checked denotes the number of checked variables.

MASKCV need loop unfolding and all bounded loops are automatically unfolded. QMVERIF, SILVER, and MASKCV require the SSA form and support automatically SSA transformation, whereas LeakageVerif does not need the SSA form.

The results are shown in Table 3, in which T.O. stands for “time-out” (6 hours) and O.O.M. stands for “out of memory.” Column 1 shows the benchmark name, column 2 shows the verification result, column 3 and column 4 (respectively, column 5 and column 6) show the verification time in seconds (s) and number of variables checked by QMVERIF without pre-conditions (respectively, with all the pre-conditions), columns 7 and 8 show the verification time and the number of variables checked by LeakageVerif, and column 9 shows the verification time of SILVER. Columns 10 and 11 show the verification time and number of variables checked by MASKCV with dominated variables (which are the variables checked at lines 9–11 in Algorithm 1 and variables of the pre-condition  $I$  used for checking first-order probing security of the program). Recall that the verification time reported in Table 3 includes all computation times for preprocessing, determining dominated variables, computing pre-conditions, and verifying first-order probing security based on pre-conditions. Since MASKCV is able to solve each benchmark in no more than 0.04 seconds, we did not report the individual computational time for each step.

We can observe that both MASKCV and QMVERIF are able to prove all of those benchmarks. However, MASKCV is significantly more efficient than QMVERIF without pre-conditions on large benchmarks that have a large number of gadget calls (e.g., AES1–AES10) but is comparable or slightly better when all pre-conditions are provided to QMVERIF. This indicates the effectiveness of our pre-condition inference approach. By comparing the number of checked variables using QMVERIF with and without user-provided pre-conditions, the number of checked variables (thus the verification time) is significantly reduced with user-provided pre-conditions. This is because a gadget call needs not be inlined if the user-provided pre-condition is satisfied by the actual parameters of the gadget call (but has to be inlined if the user-provided pre-condition is not satisfied by the actual parameters of the gadget call). Our tool MASKCV automatically infers pre-conditions of



Table 4. Results of MASKCV with and without Leveraging Dominated Variables

Name	#Gadgets	#G-Gadgets	#T-Gadgets	#G&T-Gadgets	With Dom			Without Dom		
					Time(s)	$\mathbb{I}$	#checked	Time(s)	$\mathbb{I}$	#checked
Sbox1	10	5	7	3	0.01	0	22	0.01	28	66
Sbox2	10	5	7	3	0.01	0	23	0.01	28	67
Sbox3	11	6	6	2	0.01	0	68	0.01	20	96
Sbox4	11	6	6	2	0.01	0	67	0.01	20	95
Sbox5	10	5	7	3	0.01	0	45	0.01	54	135
Sbox6	10	5	7	3	0.01	0	48	0.01	54	138
Sbox7	10	5	7	3	0.01	0	48	0.01	54	138
Sbox8	10	5	7	3	0.01	0	51	0.01	54	141
Sbox9	11	6	6	2	0.01	0	174	0.01	36	228
Sbox10	11	6	6	2	0.01	0	171	0.01	36	225
AES1	20	8	13	5	0.02	0	29	60	7,836	12,673
AES2	20	8	13	5	0.02	0	30	48	7,836	12,674
AES3	21	9	12	4	0.02	0	75	51	6,236	9,519
AES4	21	9	12	4	0.02	0	74	55	6,236	9,518
AES5	20	8	13	5	0.03	0	55	262	14,154	23,821
AES6	20	8	13	5	0.03	0	58	262	14,154	23,824
AES7	20	8	13	5	0.03	0	58	266	14,154	23,824
AES8	20	8	13	5	0.03	0	61	284	14,154	23,827
AES9	21	9	12	4	0.03	0	184	211	10,554	16,750
AES10	21	9	12	4	0.04	0	181	236	10,554	16,747
bitslicedSbox1	5	3	2	1	0.01	0	27	0.38	537	1,101
bitslicedSbox2	5	3	2	1	0.01	0	30	0.37	537	1,104

#Gadgets denotes the number of the gadgets, #G-Gadgets denotes the number of the gadgets that can generate dominant variables, #T-Gadgets denotes the number of the gadgets that can transfer dominant variables,

#G&T-Gadgets denotes the number of the gadgets that can both generate and transfer dominant variables, and  $\mathbb{I}$  denotes the size of the pre-condition of the program on which the first-order probing security is verified.

all gadgets and thus needs not check whether the inferred pre-conditions are satisfied by the actual parameters of the gadget calls, which further reduces the number of checked variables. Thus, MASKCV is comparable or slightly better than QMVERIF with all pre-conditions and significantly more efficient than QMVERIF without the pre-conditions on large benchmarks. We should emphasize that MASKCV is much easier to use since it is fully automatic, whereas QMVERIF needs user-provided pre-conditions to be efficient.

Compared with the state-of-the-art non-compositional verifiers LeakageVerif and SILVER, MASKCV is more efficient, particularly on large benchmarks (e.g., AES1–AES10). The reason is that LeakageVerif checks all observable variables after inlining gadget calls and the number of observable variables increases quickly (exponentially in the worst case) after inlining gadget calls, whereas MASKCV can directly verify composite gadgets without inlining them. SILVER fails to verify both bitslicedSbox1 and bitslicedSbox2 because it fails to construct the BDD models due to the large number of observable variables and the large size of computations.

**Answer to RQ1:** Our method is significantly more efficient than the state-of-the-art non-compositional approaches and state-of-the-art compositional approach without user-defined pre-conditions while achieving competitive efficiency compared with state-of-the-art compositional approach with user-defined pre-conditions.

**RQ2.** Table 4 presents the results of MASKCV with and without leveraging dominated variables. Column 1 shows the benchmark name; columns 2, 3, 4, and 5 show the number of gadgets (#Gadgets), the number of gadgets that can generate dominant variables (#G-Gadgets), the number of gadgets that can transfer dominant variables (#T-Gadgets), and the number of gadgets that can both generate and transfer dominant variables (#G&T-Gadgets); and columns 6, 7, and 8 (respectively, columns 9, 10, and 11) show the verification time, the size of the automatically inferred pre-condition  $\mathbb{I}$  of the program on which first-order security is checked, and the number of

```

1 power254( $\vec{x}$ ){
2    $\vec{z}$  = power2( $\vec{x}$ );
3    $\vec{z}$  = Refresh( $\vec{z}$ );
4    $\vec{y}$  = SecMult( $\vec{z}$ ,  $\vec{x}$ );
5    $\vec{w}$  = power4( $\vec{y}$ );
6    $\vec{w}$  = Refresh( $\vec{w}$ );
7    $\vec{y}$  = SecMult( $\vec{y}$ ,  $\vec{w}$ );
8    $\vec{y}$  = power16( $\vec{y}$ );
9    $\vec{y}$  = SecMult( $\vec{y}$ ,  $\vec{w}$ );
10   $\vec{y}$  = SecMult( $\vec{y}$ ,  $\vec{z}$ );
11  return  $\vec{y}$ ;
12 }
```

Fig. 7. A composite gadget defined in Sbox1.

variables checked by leveraging dominated variables (respectively, without leveraging dominated variables).

We can observe that dominated variables can significantly reduce the size of the automatically inferred pre-condition  $\mathbb{I}$  of the program, which in turn significantly reduces the number of variables that need to be checked when verifying first-order probing security of the program using the pre-condition  $\mathbb{I}$ . Indeed, all final pre-conditions  $\mathbb{I}$  of the programs by leveraging dominated variables become the empty set, because a large number of gadgets can generate dominant variables and/or transfer dominant variables from input parameters to the output encoding. It means that only the variables of simple gadgets are checked at lines through 11 in Algorithm 1 while the verification of the first-order probing security of the program based on the pre-condition  $\mathbb{I}$  is avoided because of  $\mathbb{I} = \emptyset$ . Thus, the verification time is significantly reduced by leveraging dominated variables.

**Case Study.** We exemplify the advantage of dominated variables using the composite gadget power254 shown in Figure 7, which is taken from the benchmark Sbox1. The simple gadgets power2, power4, and power16 compute an encoding  $\vec{y}$  for a given encoding  $\vec{x}$  such that  $\bigoplus \vec{y} = (\bigoplus \vec{x})^2$ ,  $\bigoplus \vec{y} = (\bigoplus \vec{x})^4$ , and  $\bigoplus \vec{y} = (\bigoplus \vec{x})^{16}$ , respectively, and the simple gadget SecMult computes an encoding  $\vec{z}$  for two given encodings  $\vec{x}$  and  $\vec{y}$  such that  $\bigoplus \vec{z} = (\bigoplus \vec{x}) \odot (\bigoplus \vec{y})$ .

The sizes of the inferred pre-conditions of the gadgets Refresh, power2, power4, power16, and SecMult by Algorithm 1 are 2, 2, 2, 2, and 4, respectively. Without leveraging dominated variables, the size of the inferred pre-condition  $\mathbb{I}$  of the composite gadget power254 by Algorithm 2 is 26, as all pre-conditions of the gadgets in the nine gadget calls are added into  $\mathbb{I}$ .

Since the gadgets Refresh and SecMult are capable of generating dominant variables, and all of the gadgets Refresh, power2, power4, and power16 can also transfer dominant variables, by leveraging dominated variables, only the pre-conditions of the gadgets power2 and Refresh and part of the pre-condition of the gadget SecMult for the first three gadget calls are added into the pre-condition  $\mathbb{I}$  of the gadget power254, whereas the pre-conditions of the gadgets power4, Refresh, power16, and SecMult in the remaining six gadget calls are not added, because  $\vec{x}_i'$  is  $\emptyset$  at line 26 of Algorithm 2. Thus, the size of the inferred pre-condition  $\mathbb{I}$  of power254 by Algorithm 2 is only 6.

Furthermore, when the gadget power254 is invoked with an actual parameter that is an dominated encoding, the calling context  $\lambda$  of the gadget power254 will be  $\lambda(\vec{x}) = \{\vec{x}\}$ . Using this additional information, the pre-conditions of the gadgets power2, Refresh, and SecMult for the first three gadget calls are not added into the pre-condition  $\mathbb{I}$  of power254 either. Thus, the pre-condition  $\mathbb{I}$  of the gadget power254 inferred by Algorithm 2 will be  $\emptyset$ .

**Answer to RQ2:** Dominated variables effectively reduce the size of pre-conditions, hence the verification time.

**Threats to Validity.** Our work focuses on cryptographic programs which, unlike general-purpose software, are structurally simple. The design of the specific program syntax tailored to the masked implementations of cryptographic algorithms has been thoroughly discussed in Section 2.1. In particular, we do not consider conditionals (e.g., if-then-else) which often induce timing side-channel leaks [71, 96], particularly under speculative execution [32, 70]. To avoid such leaks, programs often follow the constant-time principle to avoid conditionals. Therefore, it is not a real limitation and actually has been widely adopted in the literature [8, 9, 12, 17, 18, 24, 26, 30, 31, 38, 47, 69, 98]. For convenience, our tool supports bounded for-loops which are automatically and fully unfolded before verification, so we only present the core language without loops.

We did not compare with the other state-of-the-art compositional approaches (e.g., [9, 10, 18, 31]), because all existing compositional approaches (except for QMVERIFY) cannot verify the first-order security of the publicly available benchmarks considered in this work. We did not consider other benchmarks that can be handled by existing compositional approaches, as the main purpose of the current work is to handle programs that *cannot* be proved by the existing compositional approaches. Essentially, we gain generality of implementations but may sacrifice verification performance in some cases. For instance, the implementations of cryptographic algorithms only use Boolean operations, and moreover, each encoding is re-masked by using new random variables (e.g., invoking a refresh gadget) before it is used in the second place. Such Boolean programs satisfy these implicitly imposed pre-conditions in the *stronger* security notions [9, 10, 12, 16–19, 25, 26, 69], and thus can be quickly proved using the existing compositional approaches (e.g., maskVerif [10]), at the cost of the efficiency of the masked implementations.

A limitation of our work is that we only deal with first-order secure programs which may be still vulnerable against higher-order power side-channel attacks. However, we note that (1) higher-order attacks are much more difficult to launch successfully, and (2) higher-order secure programs incur excessive overhead. As a result, first-order security is more suitable especially when the efficiency is the key. Our work essentially targets at these types of applications run in, for example, resource-limited devices [20].

## 7 RELATED WORK

Along with the design of efficient masked implementations, various verification approaches have been proposed. Non-compositional approaches must inline gadget calls which would increase the size of the program and thus the verification cost. Some representative works are symbolic analysis [21, 38, 76, 77, 81, 91, 92], the SAT/SMT-based approaches SAT/SMT-based analysis [23, 47–50], BDD analysis [69], and hybrid approaches combine the symbolic analysis and SAT/SMT-based approach together [52, 54, 55, 98]. Symbolic analysis based approaches are more efficient than SAT/SMT-based approaches but only provide soundness, whereas SAT/SMT-based approaches provide both soundness and completeness in theory but limit in scalability. Hybrid approaches are aimed to bring the best of both worlds, where an SAT/SMT-based approach is applied only when the symbolic analysis fails.

Compositional approaches directly check composite gadgets, dating back to the work of Barthe et al. [9] in which stronger security notions and a sound proof system are proposed. Basically, when a return encoding is used as actual parameters multiple times, their approach directly sums up all possible numbers of observable variables, leading to an inaccurate conclusion. We detail in Appendix 8 why their proof system fails to prove the running example in Figure 5. Along this direction, various approaches have been proposed to check simple gadgets [10, 12, 26, 38, 69] with

respect to the security notions of Barthe et al. [9]. SILVER [69] supports probing security but fails to prove first-order probing security in our experiments. Belaïd et al. [17–19] presented alternative composition approaches of Barthe et al. [9] via matrix analysis; however, it assumes that all gadgets are ISW gadgets [12] but some efficient gadgets that are non-ISW, such as the first-order masked AND gadget [20]. Cassiers et al. [30, 31] proposed an alternative stronger security notion PINI such that gadgets composed by PINI gadgets are still PINI. Even though all of these compositional approaches support higher-order security, only bitwise logical operations are focused, except for that of Coron [38], and the pre-conditions in those security notions are fixed, which cannot be fulfilled by some efficient gadgets, where that of Coron [38] are semi-automatic. Our approach is wider (e.g., arithmetic operations and non-ISW gadgets) for the first-order security of composite gadgets. The approach proposed by Gao et al. [53] is applicable to general first-order masked implementations, but it requires user-defined pre-conditions to be efficient. Our work bridges the gap that there lacks an efficient compositional approach that is applicable to general gadgets without user-defined pre-conditions.

Mitigating techniques have been studied to eliminate power side-channel leaks [2, 9, 13, 17, 25, 46, 92]; however, they either do not use formal verification to provide guarantees [2, 13] or rely on formal verification [9, 17, 25, 46, 92]. We focus on verification, but it could also be extended to eliminate power side-channel leaks, similar to other works [9, 17].

We note that there are other types of side-channel attacks such as timing side-channel attacks which have been widely studied, including detection [7, 29, 79, 82, 83], verification [4, 6, 22, 33, 42, 44, 45, 61], and mitigation [75, 95, 96]. The more recent work focuses on timing side-channel leaks introduced by micro-architectural features [34, 43, 59, 60, 70, 73] and (JIT) compilation [27, 28, 42, 43, 85], and network-based side channel leaks [65, 66, 88] which contain a large number of observable aspects (e.g., the time, size, and direction of each packet). As different types of side-channel attacks have their own characteristics, they are orthogonal to our work. However, there are other power leakage models and security notions [10, 16, 23, 51, 78, 92] which are outside of the scope of this work. Finally, we remark that this work considers the first-order probing security of programs written in our domain-specific language while secure programs may become insecure after compilation [92], the same as in the timing side-channel security setting.

## 8 CONCLUSION

We proposed a novel approach and algorithms to infer pre-conditions for compositional verification of generic and efficient masked implementations. We implemented our approach in a tool and conducted extensive experiments on publicly available benchmarks. It significantly outperforms the state of the art on masked implementations of full AES when no pre-conditions are provided by users.

## APPENDIX

### A.1 The Type System of Barthe et al. [9]

We explain why the type system of Barthe et al. [9] fails on XORMULTI.

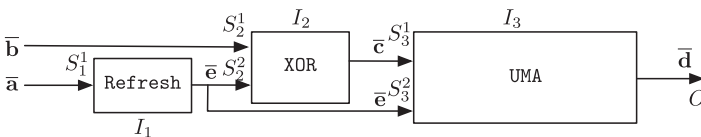


Fig. 8. Input-output relation of XORMULTI.

Figure 8 shows the input-output relation of XORMULTI. It is easy to conclude that Refresh is 1-SNI, XOR is 1-NI, and UMA is 1-SNI. Let  $O$  denote the set of observed variables from the output of XORMULTI. Let  $I_i$  denote the set of observed internal variables from the  $i$ -th gadget call. Let  $S_i^j$  denote the number of input shares needed to simulate further internal nodes. To prove whether XORMULTI is 1-NI, the global constraint is  $|I_1| + |I_2| + |I_3| + |O| \leq 1$ . Let us get started from right to left in Figure 8:

- (1) The side condition  $|O| + |I_3| \leq 1$  is satisfied. As UMA is 1-SNI, we can obtain  $|S_3^1| \leq |I_3|$  and  $|S_3^2| \leq |I_3|$ .
- (2) The side condition  $|S_3^1| + |I_2| \leq 1$  is satisfied. As XOR is 1-NI, we can obtain  $|S_2^1| \leq |I_2| + |S_3^1|$  and  $|S_2^2| \leq |I_2| + |S_3^2|$ .
- (3) The side condition  $|I_1| + |S_3^2| + |S_2^2| \leq 1$  cannot be proved through the constraints, as we can only conclude that  $|I_1| + |S_3^2| + |S_2^2| \leq |I_1| + 2|I_3| + |I_2|$ . The proof terminates here.

From the preceding, it is easy to conclude that Barthe's work can prove neither 1-NI nor 1-SNI of XORMULTI.

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